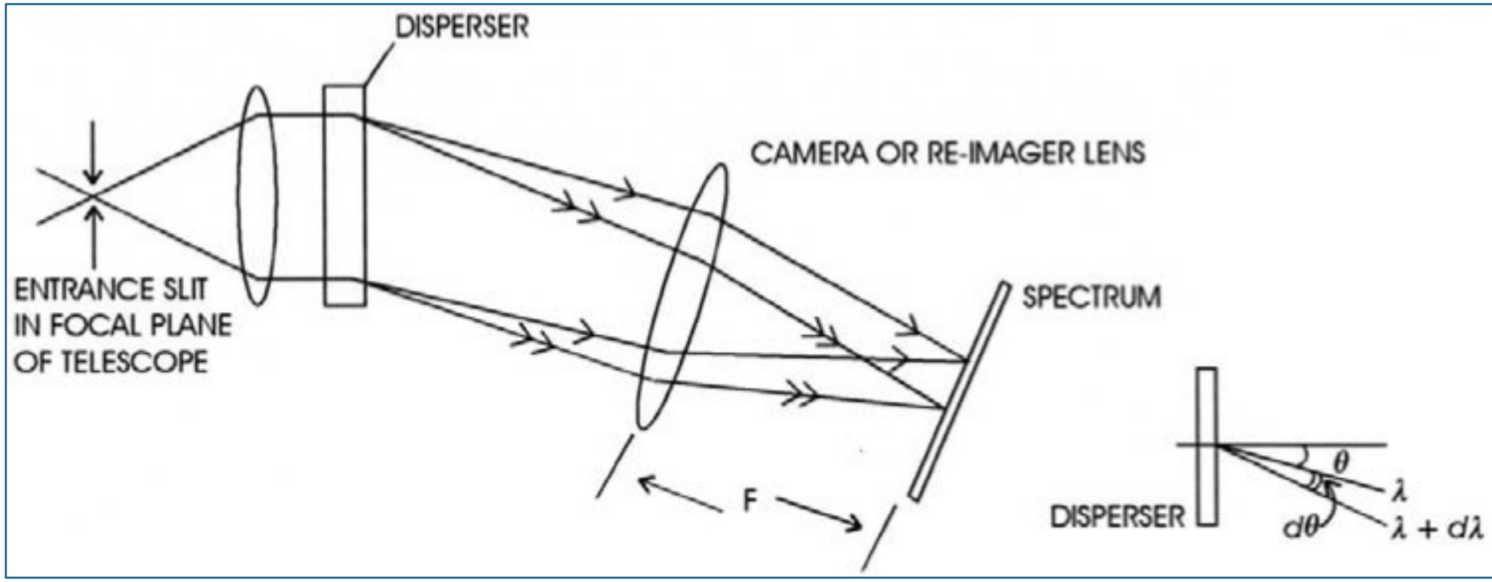
Mclean seminar sec.5.2-5.2.4

2024/06/28

M1 Kensho Tanaka

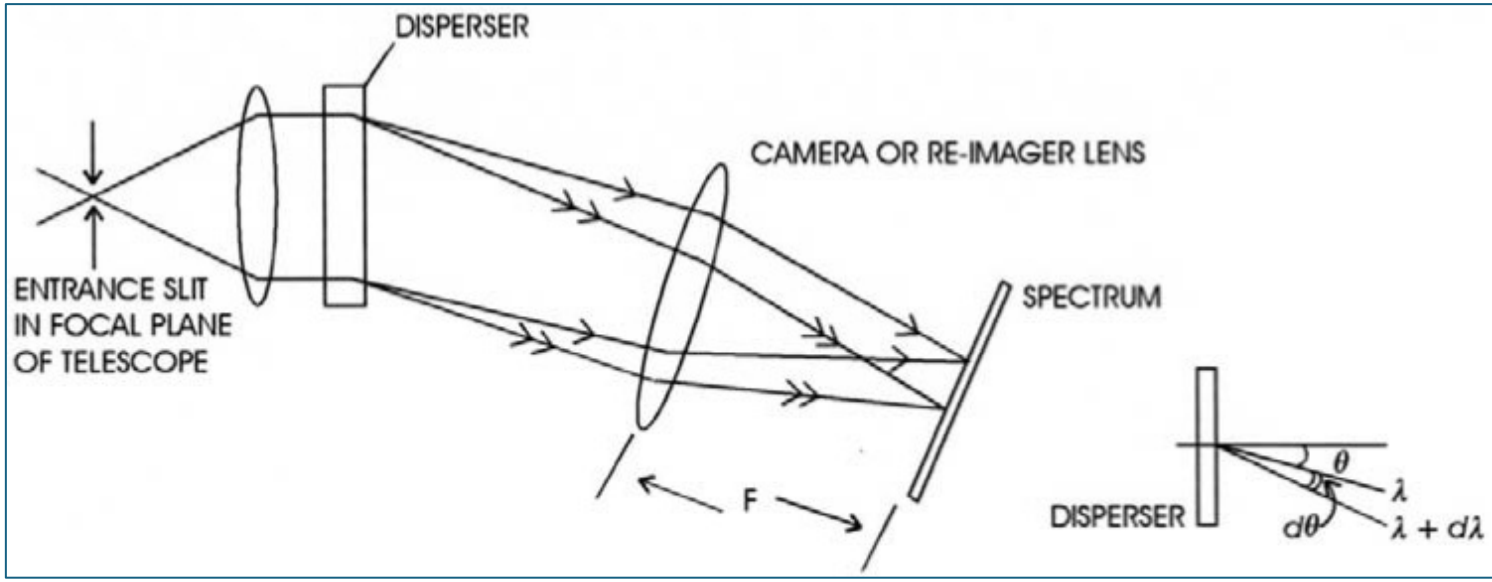
5.2 Spectrometers

- All spectrometers have essentially the same basic design.
- But many different implementations are possible depending on the constraints and choice of **spectral disperser**.



5.2 Spectrometers

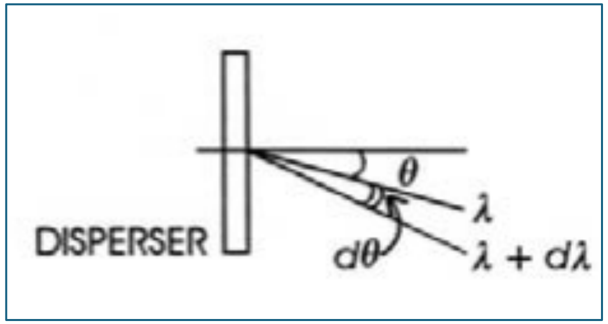
- The important quantities
  - the resolving power (R)
  - the slit width
  - the diameter of the collimated beam
  - the sampling or matching of the slit width to the detector pixels
  - the resulting f/number of the camera system



5.2.1 Resolution and dispersion

- Angular Dispersion (AD)
  - the rate of change of the dispersed angle of the beam with respect to wavelength

$$AD = \frac{d\theta}{d\lambda}$$



- Linear Dispersion (LD)
  - relates an interval of length (dx in millimeters) along the spectrum to a wavelength interval (mm/A)

$$LD = \frac{dx}{d\lambda} = \frac{dx}{d\theta}\frac{d\theta}{d\lambda} = f_{cam}AD$$

5.2.1 Resolution and dispersion

- Resolving Power
  - the ability to distinguish two wavelengths separated by a small amount $\Delta\lambda = \lambda_2 - \lambda_1$

$$R = \frac{\lambda}{\Delta\lambda}$$

  - Resolution is often stated

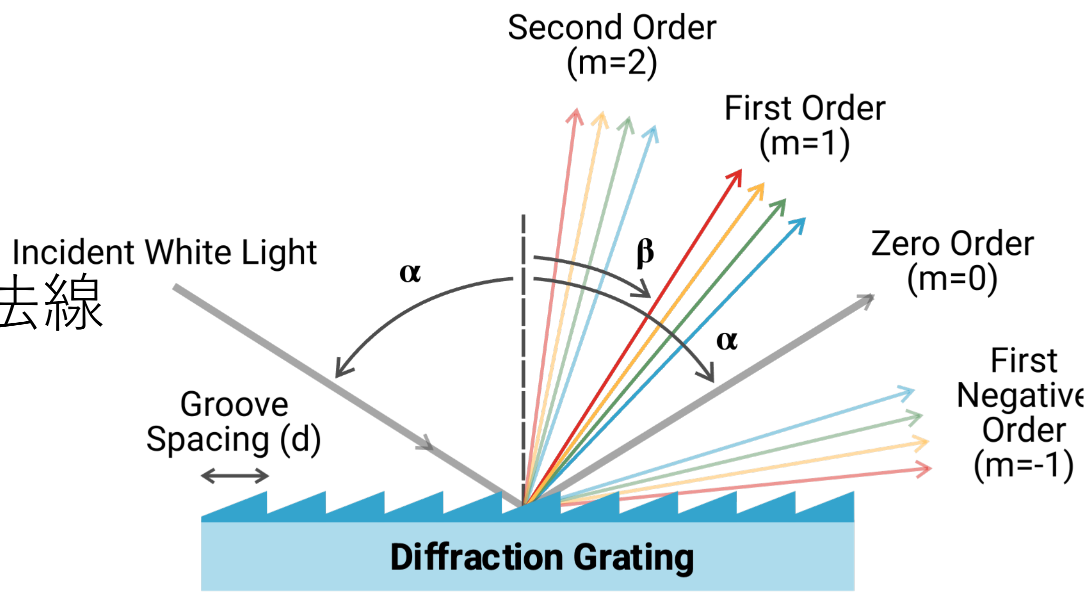
$$\frac{1}{R} = \frac{\Delta\lambda}{\lambda} = \frac{V}{c}$$

  - where the non-relativistic Doppler formula
  - (e.g. R = 10000 > V = 0.0001c = 30km/s)

5.2.2 Diffraction gratings

- The usual dispersing element is a diffraction grating and the general grating equation is
$$m\lambda = d(\sin i + \sin\theta)\cos\gamma$$
  - d : the spacing of adjacent grooves or slits
  - i (represented by α) : the angle of incidence of the collimated beam
  - θ (represented by β) : the angle of the emergent diffracted beam
  - γ : the angle out of the normal plane (法線平面) of incidence (usually 0°)
  - m : an integer called the "order" of interference
- It can apply when the grating is used in transmission or in reflection

Grating and Diffraction Orders



(https://www.meetoptics.com/academy/groove-density)

5.2.2 Diffraction gratings

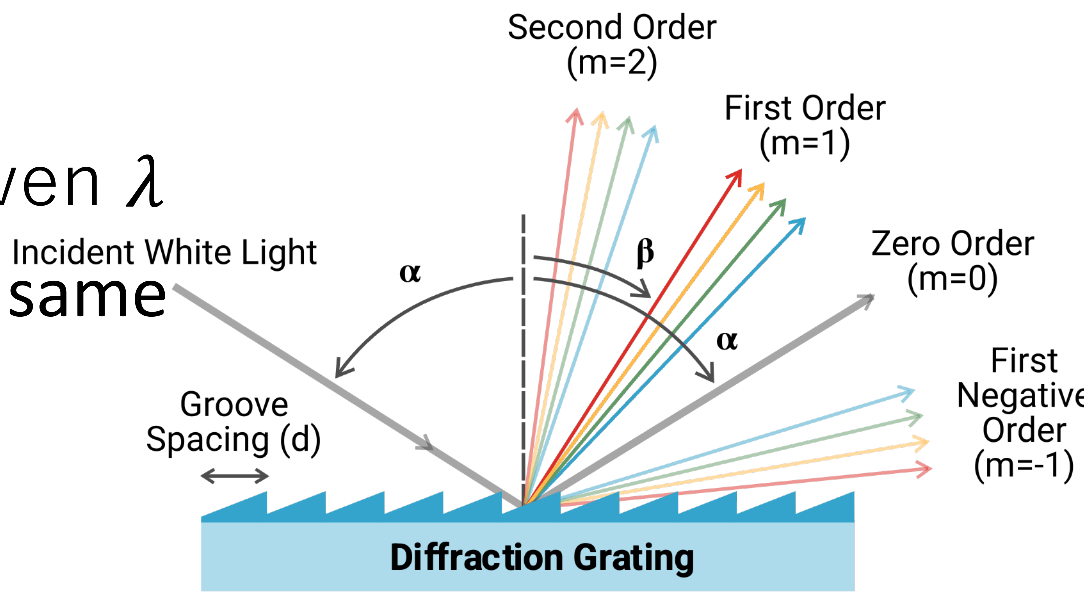
- The alternative form

$$m\lambda = d(\sin i + \sin\theta)\cos\gamma$$

$$\text{AD} = \frac{d\theta}{d\lambda} = \frac{m}{d\cos\theta\cos\gamma} = \frac{\sin i + \sin\theta}{\lambda\cos\theta}$$

- AD is determined by i and θ for a given λ
- Many combinations of m and d yield the same AD provided the grating angles remain unchanged

Grating and Diffraction Orders



(https://www.meetoptics.com/academy/groove-density)

5.2.2 Diffraction gratings

- Typical "first-order gratings" (m ~ 1) have 300-2,400 grooves or lines/millimeter.
- the number of lines per millimeter is given by T = 1/d.

- $cos\theta \sim 1$ and slowly varying
  - AD is almost constant
  - the relationship between position and wavelength on the detector (Equation (5.8), the upper equation) is approximately linear.

$$LD = \frac{\boldsymbol{dx}}{\boldsymbol{d\lambda}} = \frac{dx}{d\theta}\frac{d\theta}{d\lambda} = f_{cam}AD$$

$$\text{AD} = \frac{d\theta}{d\lambda} = \frac{m}{d\cos\theta\cos\gamma} = \frac{\sin i + \sin\theta}{\lambda\cos\theta}$$

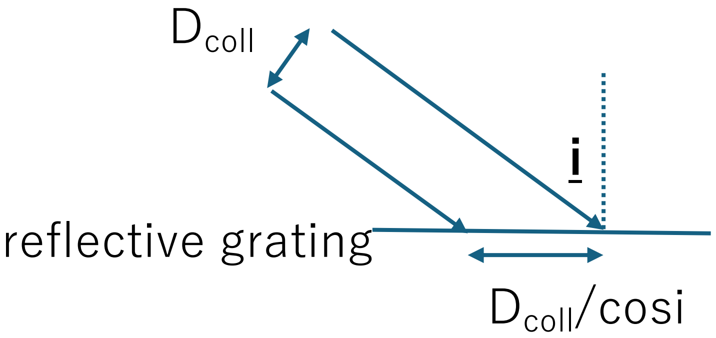
5.2.2 Diffraction gratings

- "Echelle" gratings
  - Coarse-ruled reflection gratings (**large d, small T**) can achieve high angular dispersion by making $i\ and\ \theta$ very large.
  - groove densities : T = 20-200 lines/millimeter
  - m : 10-100
  - this results in severe overlap of orders unless a second disperser of lower resolving power at right angles to the first is used to "separate" the orders.

5.2.2 Diffraction gratings

- In a standard astronomical spectrograph, the light emerging from the slit
  - is collimated into the parallel beam of diameter D_{coll}
  - directed onto a reflection grating at an angle of incidence i
  - so that illuminated length is $D_{coll}/\cos i$

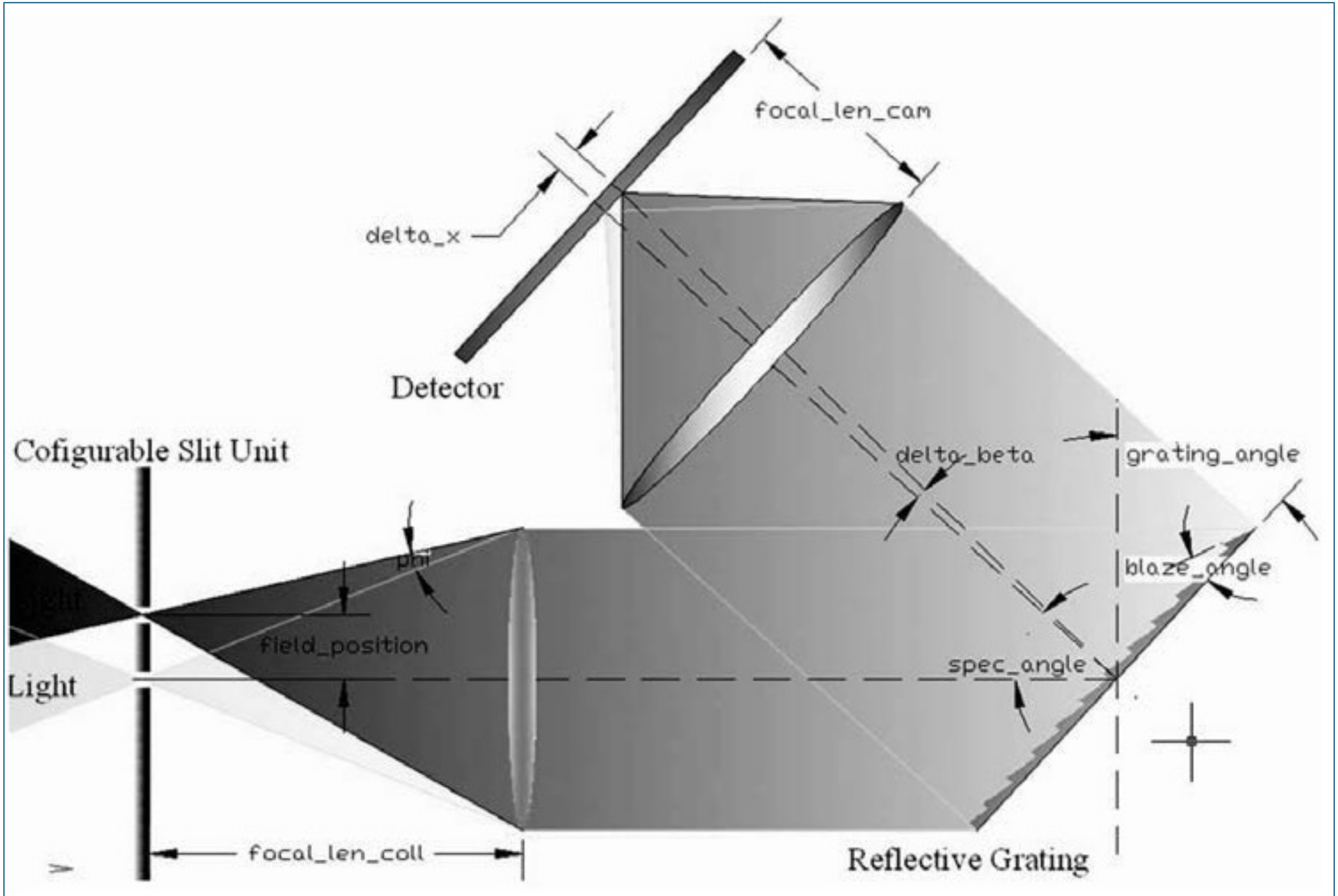
- **The magnification between the slit and the detector :** f_{cam}/f_{coll}



5.2.2 Diffraction gratings

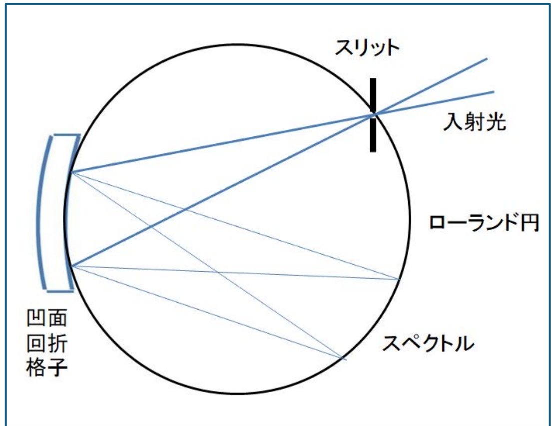
- It is much more convenient
  - to keep **the collimator** and **the camera optics** in a **fixed** position
  - to **allow** some limited motion of **gratings**
    - than to **fix the grating** and require **the camera optics to move in an arc** (弧状に動かす) to pick up different parts of the diffracted beam.

- Two optical axes
  - defined by collimator
  - defined by the camera
  - These axes intersect on the reflection grating.

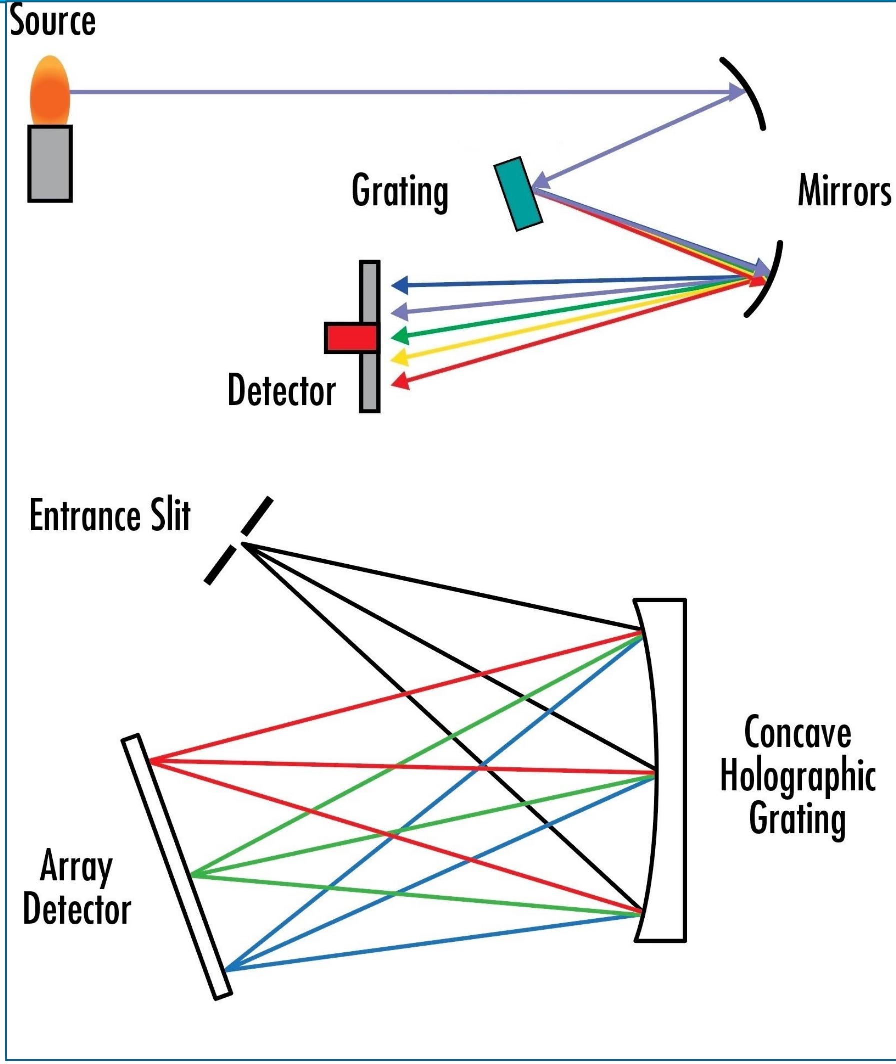

focal_len_cam
delta_x
Detector
Cofigurable Slit Unit
delta_beta
grating_angle
phi
blaze_angle
field_position
Light
spec_angle
focal_len_coll
Reflective Grating

5.2.2 Diffraction gratings

- The example of concave gratings (lower picture)
  - this approach is ideally suited for the **far-ultraviolet** where transmission lenses are difficult to obtain.
- Rowland Circle
  - the circle which has the same curvature of concave grating



(https://astro-dic.jp/rowland-circle/)



(https://www.edmundoptics.jp/)

5.2.2 Diffraction gratings

- A grating produces a different magnification in the dispersion direction than at right angles to the dispersion.
- The “anamorphic” magnification factor describes this effect and is found by determining the change in θ for a change in i

$$\frac{d\theta}{di} = \left|\frac{\cos i}{\cos\theta}\right|$$

- the size of the slit image (Δx) at the detector becomes

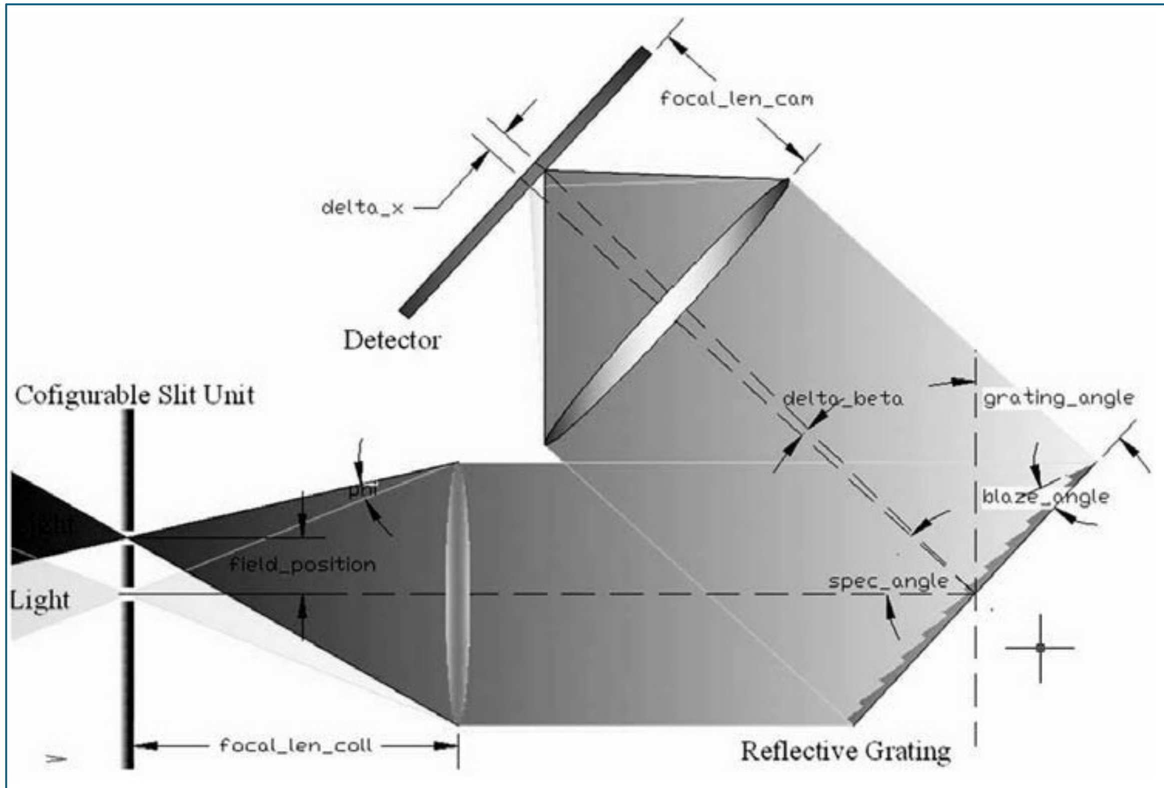
$$\Delta x = w\ \frac{\cos i}{\cos\theta}\frac{f_{cam}}{f_{coll}}$$

- w : slit width

5.2.2 Diffraction gratings

$$\Delta x = w \frac{\cos i}{\cos \theta} \frac{f_{cam}}{f_{coll}}$$

- If $i < \theta$
  - the grating normal is more nearly pointed at the collimator
  - the image of the slit is wider in the spectral direction
    - pixel sampling is better.
    - the resolution is reduced.



5.2.2 Diffraction gratings

- If the grating is to accept all the light from the collimator then it follows that the ruled width of the grating (W)

$$W = D_{coll} / \cos i$$

- In the diffraction-limited case

$$R = mN = \frac{mW}{d} = \frac{W(\sin i + \sin\theta)}{\lambda}$$

  - N : the total number of grooves illuminated

- In practice, spectrometers are usually slit width limited or seeing-limited.

5.2.2 Diffraction gratings

- In practice, spectrometers are usually slit width limited or seeing-limited.
- If the slit width ~ seeing disk, $\theta_{see} = \lambda/D_{tel}$

$$R = \frac{W(\sin i + \sin\theta)}{\theta_{see} D_{tel}}$$

- As D increases, the resolving power decreases, unless W gets larger.

$$\theta_{see} = p \times \theta_{pix}$$

- p : the number of pixels across the slit image

$$R = \frac{(\sin i + \sin\theta)}{\cos i} \frac{D_{coll}}{D_{tel}} \frac{206265}{p\theta_{pix}}$$

5.2.2 Diffraction gratings

$$R = \frac{(\sin i + \sin\theta)}{\cos i}\frac{D_{coll}}{D_{tel}}\frac{206265}{p\theta_{pix}}$$

- the trade-offs of size(p) vs resolution (R)
- To maintain R, as the telescope diameter increases, the spectrograph gets larger (D_coll gets larger).

5.2.2 Diffraction gratings

- The intensity distribution (I) from an ideal grating can be derived by expanding the analysis of wave interference from single and double slits to N slits (Chapter.2)

$$I = A_0^2 \frac{\sin^2 \beta}{\beta^2} \frac{\sin^2 N\gamma}{\sin^2 \gamma}$$

- $\gamma = \pi d \sin\theta / \lambda$: the phase difference between adjacent of slits of separation d
  - contribution of N slits
- $\beta = \pi b \sin\theta / \lambda$: the phase difference from the center of on slit (width : b) to its edge
  - represents the case of single slit

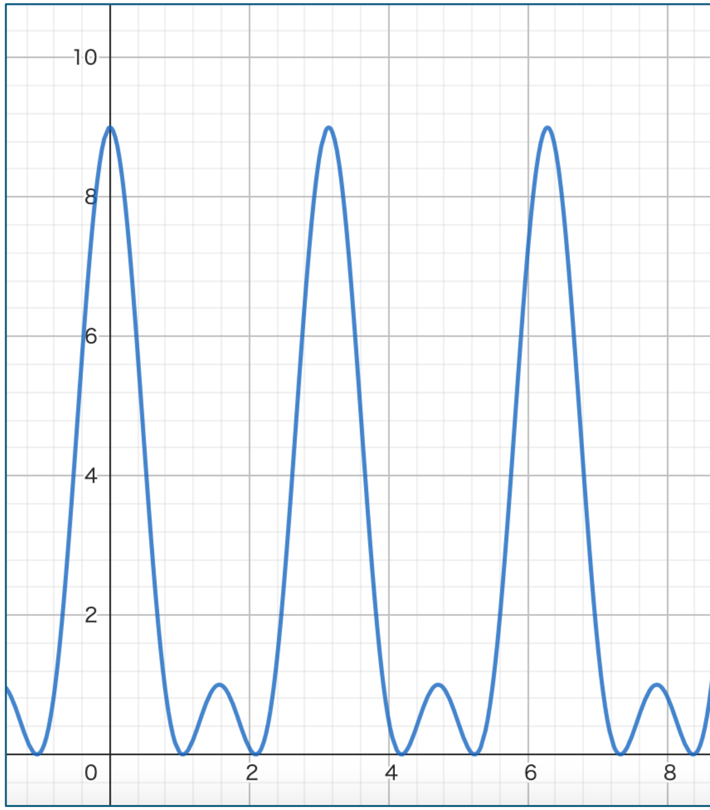
5.2.2 Diffraction gratings

$$I = A_0^2 \frac{\sin^2\beta}{\beta^2}\frac{\sin^2 N\gamma}{\sin^2\gamma}$$

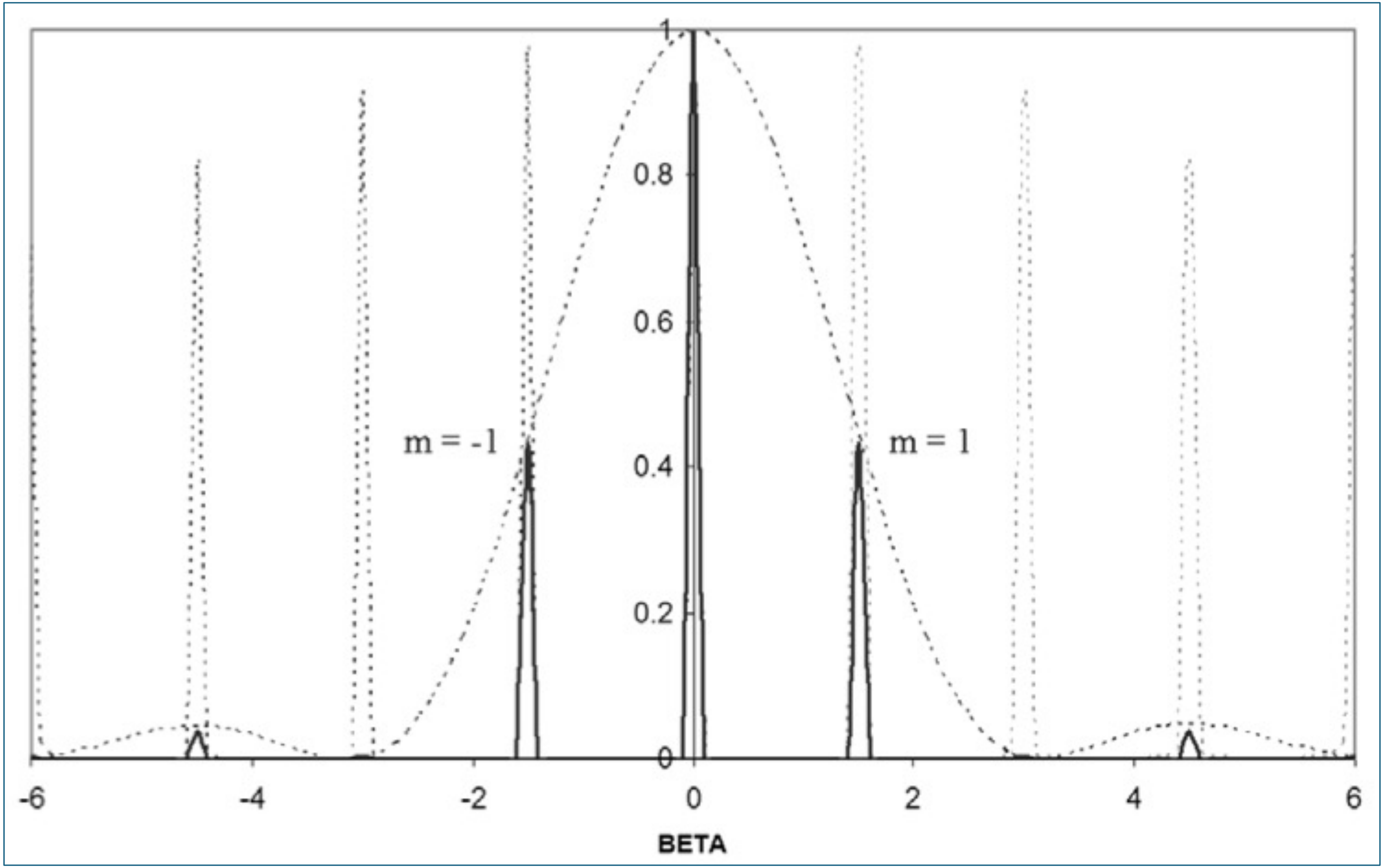
- $\frac{\sin^2 N\gamma}{\sin^2\gamma}$ $(\gamma = \pi d \sin\theta / \lambda)$
  - strong maximum values equal to N^2
  - $\gamma = 0, \pi, 2\pi \ldots > \mathrm{d}\sin\theta = m\lambda$
  - Secondary maxima between orders are strongly suppressed.
- $\frac{\sin^2\beta}{\beta^2}$ $(\beta = \pi b \sin\theta / \lambda)$
  - first minimum value (0) when $b\sin\theta = \lambda$

Drastically reducing the intensity at $m = \pm 1$

$$y = \frac{\sin^2 3x}{\sin^2 x}$$



(produced by GeoGebra)



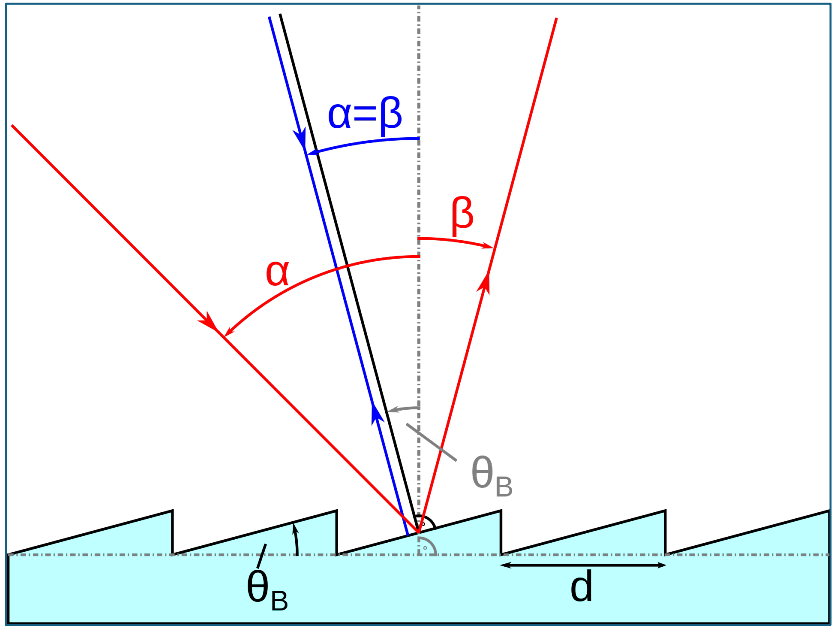
1
0.8
0.6
0.4
0.2
0
m = -1
m = 1
-6
-4
-2
0
2
4
6
BETA

5.2.2 Diffraction gratings

- For any given order of diffraction, except $m = 0$, different wavelengths are diffracted **at different angles**.
  - > producing a spectrum
- when the single-slit diffraction pattern maximizes the diffracted intensity at zero order.
  - > no dispersion occurs. first/second order spectra are very faint.
- We need to be able **to “shift”** the peak of this envelope to $m = 1$.
- This is possible for reflection gratings.

5.2.2 Diffraction gratings

- by tilting the facets of a reflection grating through an angle **θ_B (Blaze angle)** with respect to the plane of the grating surface,
  - it is possible to maximize the grating efficiency in the direction in which light would have been reflected in the absence of diffraction.
- $i(\alpha) + \theta(\beta) = 2\theta_B$ (most efficient)
- $i(\alpha) - \theta(\beta) = \phi$ (definition of spectrograph angle)
- $m\lambda_B = 2d \sin\theta_B \cos(\phi/2)$



(https://astro-dic.jp/blaze-angle/)

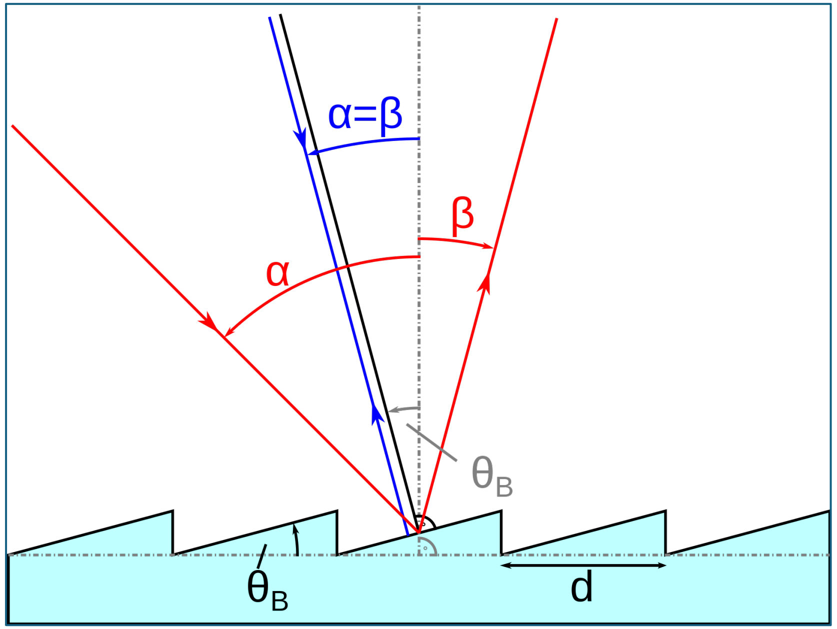
5.2.2 Diffraction gratings

$$m\lambda_B = 2d \sin\theta_B \cos(\phi/2)$$

- **“Littrow” condition**
  - $\phi = 0$
  - $i = -\theta = \theta_B$ the incident and diffracted angles measured relative to the grating normal are equal to each other

$$m\lambda_B = 2d \sin\theta_B$$

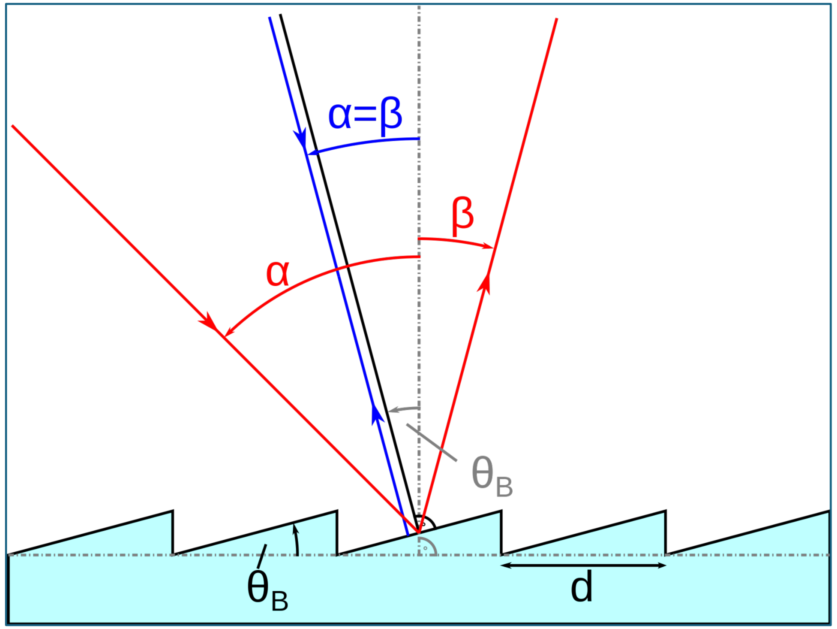
$$R = \frac{2D_{coll} \tan\theta_B}{p\theta_{pix}D_{tel}}$$



(https://ja.wikipedia.org/wiki/ブレーズド回折角)

5.2.2 Diffraction gratings

- The only way to work in the Littrow condition is with a central obscuration in the optics.
- Alternatively, one can use the “near” Littrow condition by moving off by 10° - 20° or the **“quasi”** Littrow condition by going out of the plane ($\gamma > 0°$).
  - Grating efficiency drops **rapidly** as the angle away from Littrow grows.
  - the drop is very **slow** for quasi-Littrow mode.
    - the slit image is tilted.



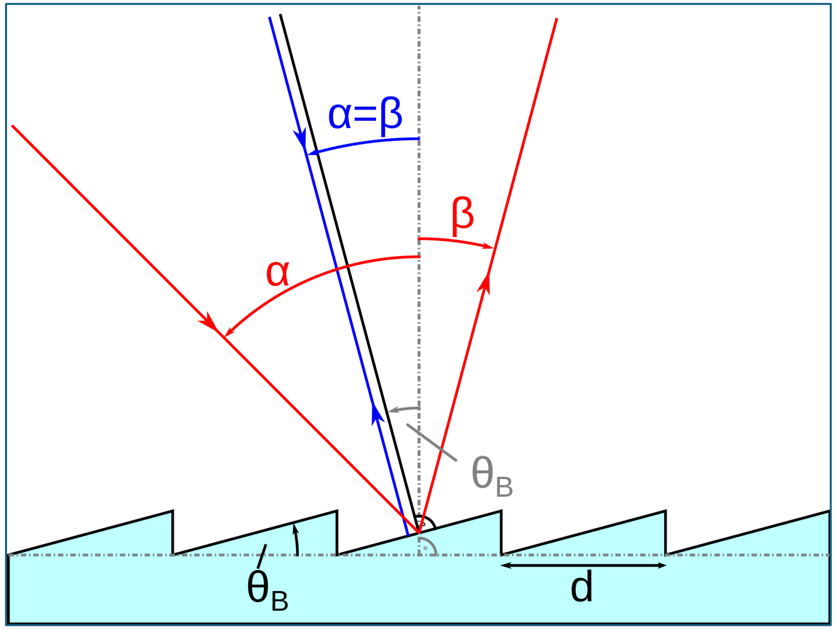
(https://ja.wikipedia.org/wiki/ブレーズド回折角)

5.2.2 Diffraction gratings

- the tilted angle χ

$$\tan\chi = \frac{\tan\gamma\,(\sin i + \sin\theta)}{\cos\theta}$$

- when $\tan\theta_B = 2$ echelle grating, $\tan\chi = 4\tan\gamma$
  - $\gamma = 5° > \chi = 19.3°$
- there is also a change $\Delta\chi$ in this angle across an order
  - the higher the order, the smaller the change



(https://ja.wikipedia.org/wiki/ブレーズド回折角)

5.2.2 Diffraction gratings

- for a given pair of incident and diffraction angles the grating equation is satisfied for all λ for which **m** is an integer.
- There are two wavelengths in successive orders

$$m\,\lambda' = (m+1)\lambda$$

- the wavelength difference $\lambda' - \lambda$ is called Free Spectral Range **(FSR)**

$$\Delta\lambda_{FSP} = \lambda/m$$

- The two wavelengths are diffracted in the same direction
    - requires either an “order sorter” filter or a cross-disperser element(when m is large)
    - order sorting filters must be carefully chosen to cut on and off sharply to prevent order overlap (Section 5.4.3)

5.2.2 Diffraction gratings

- Grating efficiency is difficult to calculate.
- Peak efficiency should occur at λ_B when m = 1 and then declining peaks should occur at λ_B/m in subsequent orders.

5.2.2 Diffraction gratings

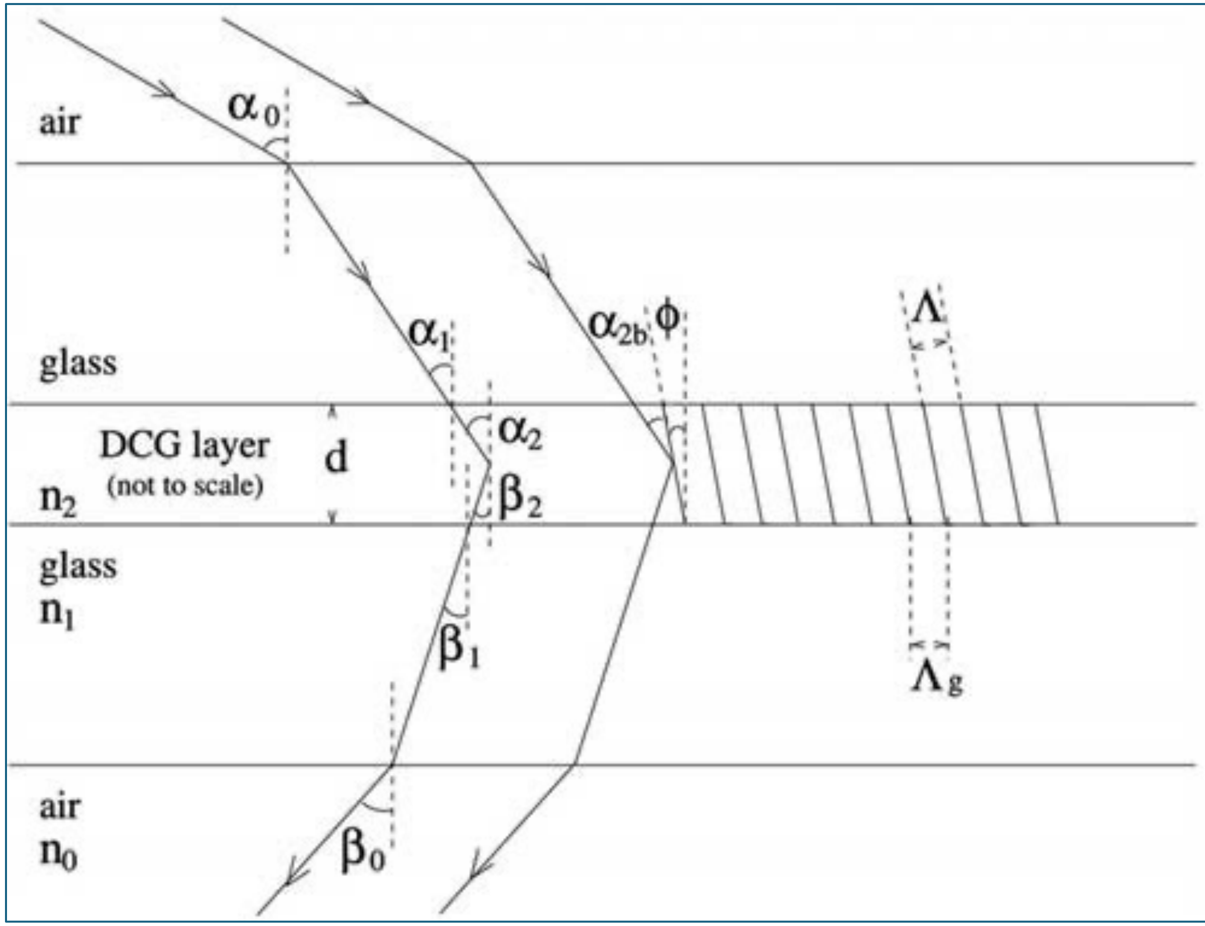
- Volume Phase Holographic (VPH) grating
  - a new technology for grating fabrication
  - an optical substrate in which the refractive index varies periodically throughout the body of the grating.
  - grating body is made from a thin(3-30μm) slab of **dichromated gelatine(DCG)** trapped between glass plates

5.2.2 Diffraction gratings

- Light passing through a VPH transmission grating

$$m\lambda = n_i \Lambda_g (\sin\alpha_i + \sin\beta_i)$$

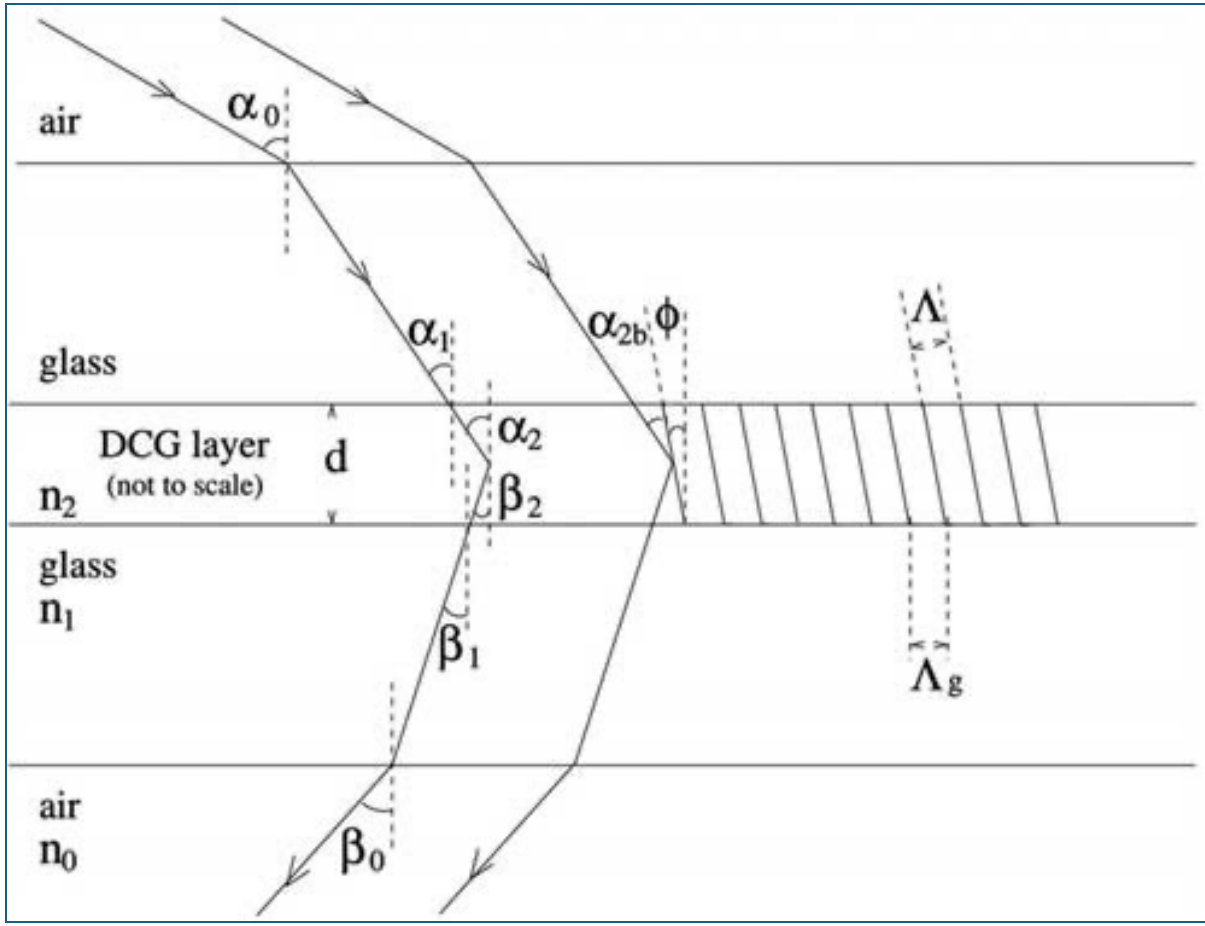
- m : an integer of the order
- n_i : the refractive index of the medium
- Λ_g : the grating period (groove spacing)



5.2.2 Diffraction gratings

$$m\lambda = n_i \Lambda_g (\sin\alpha_i + \sin\beta_i)$$

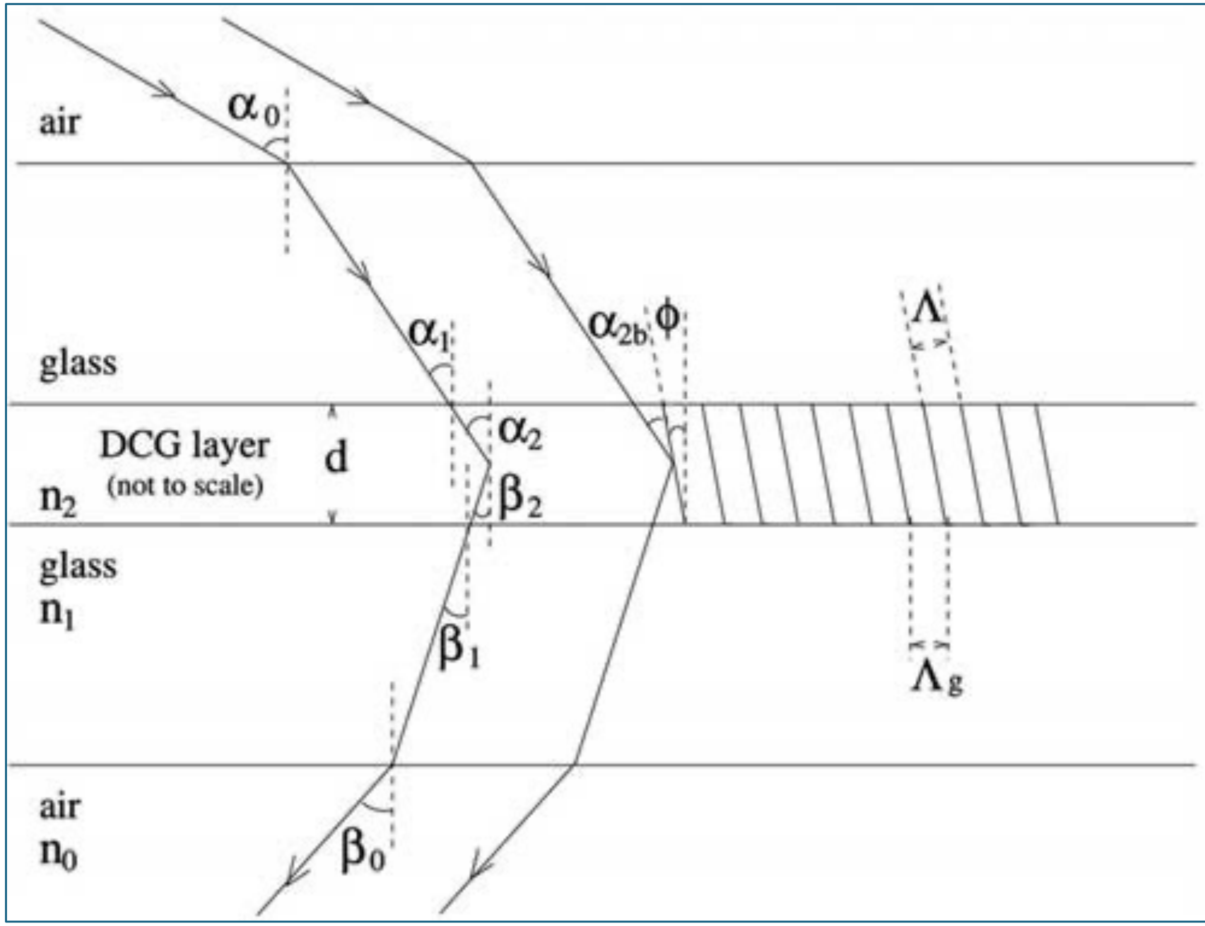
- High diffraction efficiency can occur when light is effectively reflected from the plane of the fringes ($\beta_2 + \phi = \alpha_2 - \phi$).
  - the same as Bragg diffraction
    $$m\lambda = 2n_2 \Lambda_g \sin\alpha_{2b}$$
- n_2: the refractive index of DCG layer
- $\alpha_{2b} = \alpha_2 - \phi$: Bragg angle



5.2.2 Diffraction gratings

$$m\lambda = 2n_2\Lambda_g \sin\alpha_{2b}$$

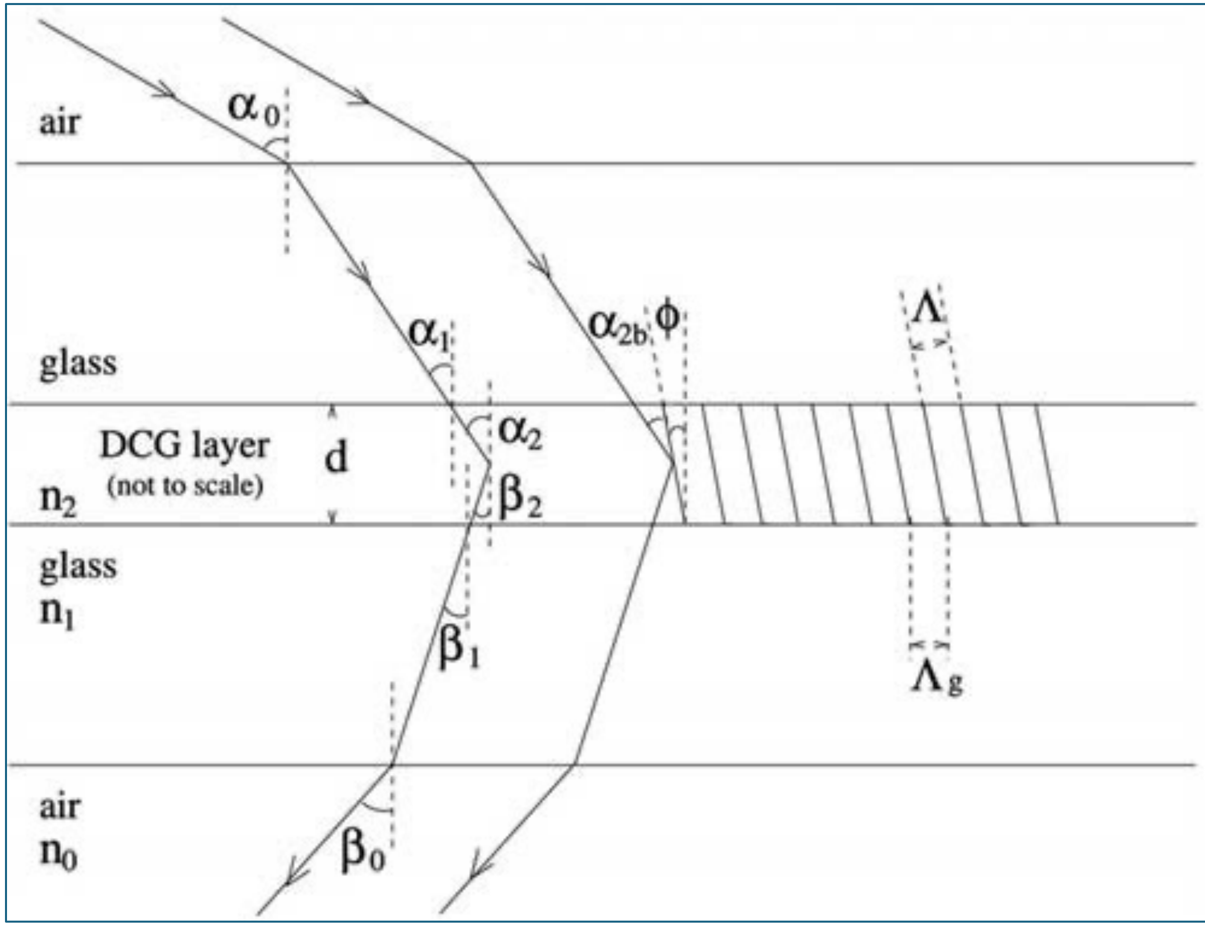
- At wavelengths sufficiently displaced from the Bragg condition, there is no diffraction.
- Diffraction efficiency depends on the semi-amplitude of the refractive index modulation (Δn_2) and the grating thickness (d).



5.2.2 Diffraction gratings

$$m\lambda = 2n_2\Lambda_g \sin\alpha_{2b}$$

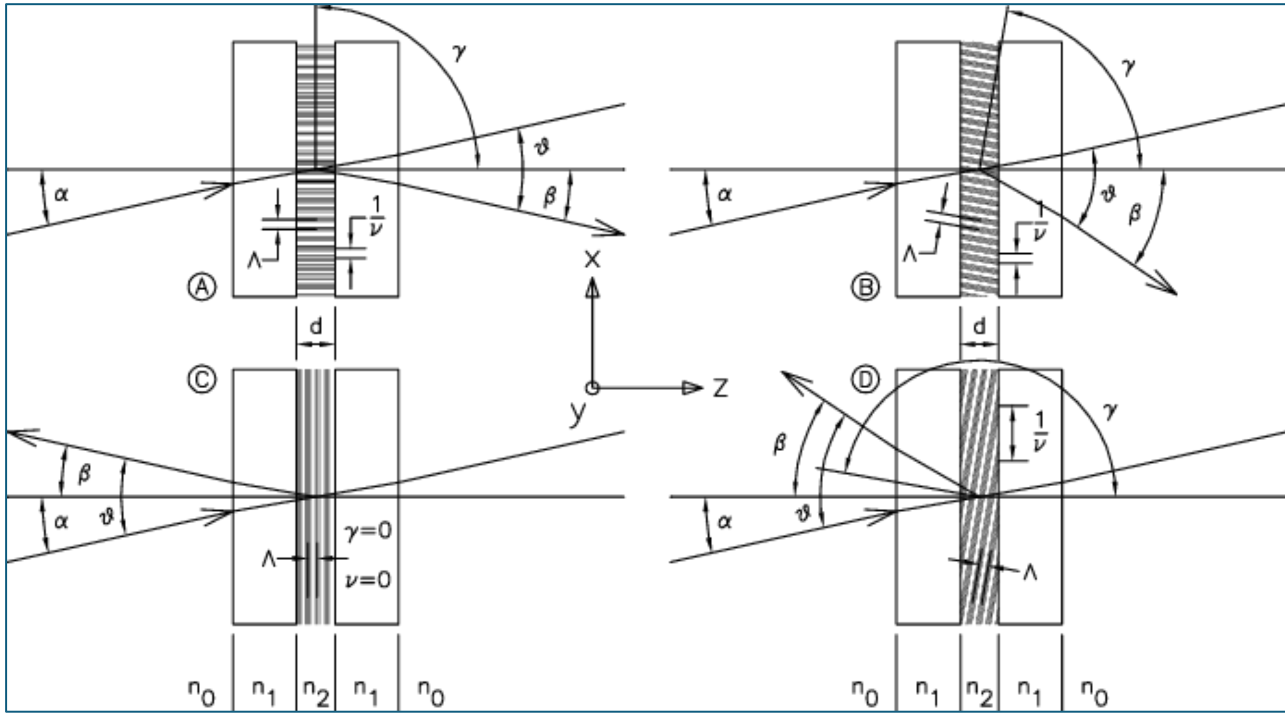
- The DCG holds a fringe pattern generated by holography
  - which provides planes of constant refractive index separated by a length $\Lambda = 1/v_g$
- Index variations are the result of **density variations** which are trapped into the material by exposure to light (the fringe pattern).



5.2.2 Diffraction gratings

$$m\lambda = 2n_2\Lambda_g \sin\alpha_{2b}$$

- One form for the refractive index
  $n_2(x,z)$
  $= n_2 + \Delta n_2 \cos[2\pi v_g(x \sin\gamma + z \cos\gamma)]$
- gives the variation in the x, z plane where z is the optical axis through the VPH, and γ is the angle between the normal to the planes and the z-axis.
- v_g : line densities 300-6000lines/mm
- Δn_2: 0.02-0.1



(Tunable Gratings: Imaging the Universe in 3-D with Volume-Phase Holographic Gratings Barden et al. 1999)

5.2.2 Diffraction gratings

- Because of Bragg condition, it is necessary to **articulate the camera to a new angle** to tune to a new wavelength.

- This area of technology is receiving a great deal of research attention, in part because of the possibility of making VPH gratings **in large sizes.**

- A VPH is used in the 6dF(six degree Field) spectrograph
- The use of “immersion” gratings
  - in which the grating surface is coupled to or embedded in a prism
  - so that n is returned to the grating equation $m\lambda = 2dn\sin\theta$

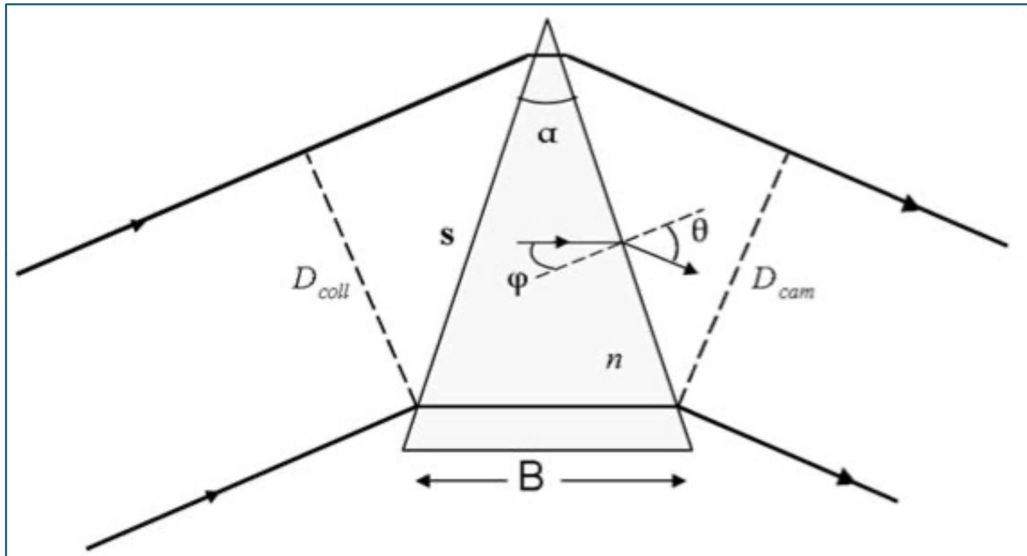
5.2.3 Prisms

- Prism is used
  - as a primary disperser in (usually) low-resolution instruments
  - as a cross-disperser in high-resolution echelle spectrographs

◯ cheaper and easier to make

◯ no interference effects and no overlapping orders to handle

× high resolving power is very difficult

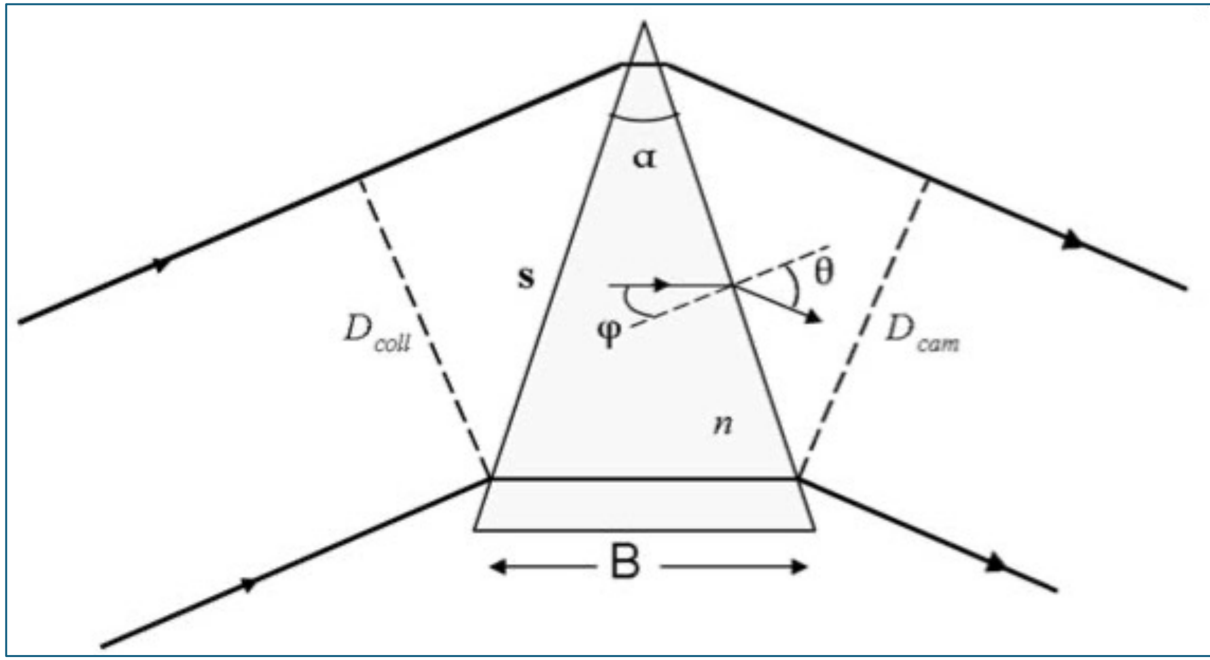


5.2.3 Prisms

- Angular dispersion

$$\frac{d\theta}{d\lambda} = \frac{d\theta}{dn}\frac{dn}{d\lambda} = \frac{B}{D_{cam}}\frac{dn}{d\lambda}$$

- $\frac{dn}{d\lambda}$
  - the wavelength dependency of the refractive index
- $\frac{d\theta}{dn}$
  - derived by differentiating Shell's law
  - $\frac{d\theta}{dn} = [2s\sin\left(\frac{\alpha}{2}\right)/scos\theta]$
  - $2s\sin\frac{\alpha}{2} = B$
  - $s\cos\theta = D_{cam}$



5.2.3 Prisms

- Angular dispersion

$$\frac{d\theta}{d\lambda} = \frac{d\theta}{dn}\frac{dn}{d\lambda} = \frac{B}{D_{cam}}\ \frac{dn}{d\lambda}$$

- The resolving power of a prism

$$R\ = B\left(\frac{dn}{d\lambda}\right) \quad [D_{cam} d\theta = \lambda]$$

- for a slit-limited instrument

$$R = \frac{\lambda}{\theta_{res} D_{tel}}\ B\frac{dn}{d\lambda}$$

- n usually increases steeply toward shorter wavelengths.
- the blue end of a prism is more spread out than the red end.

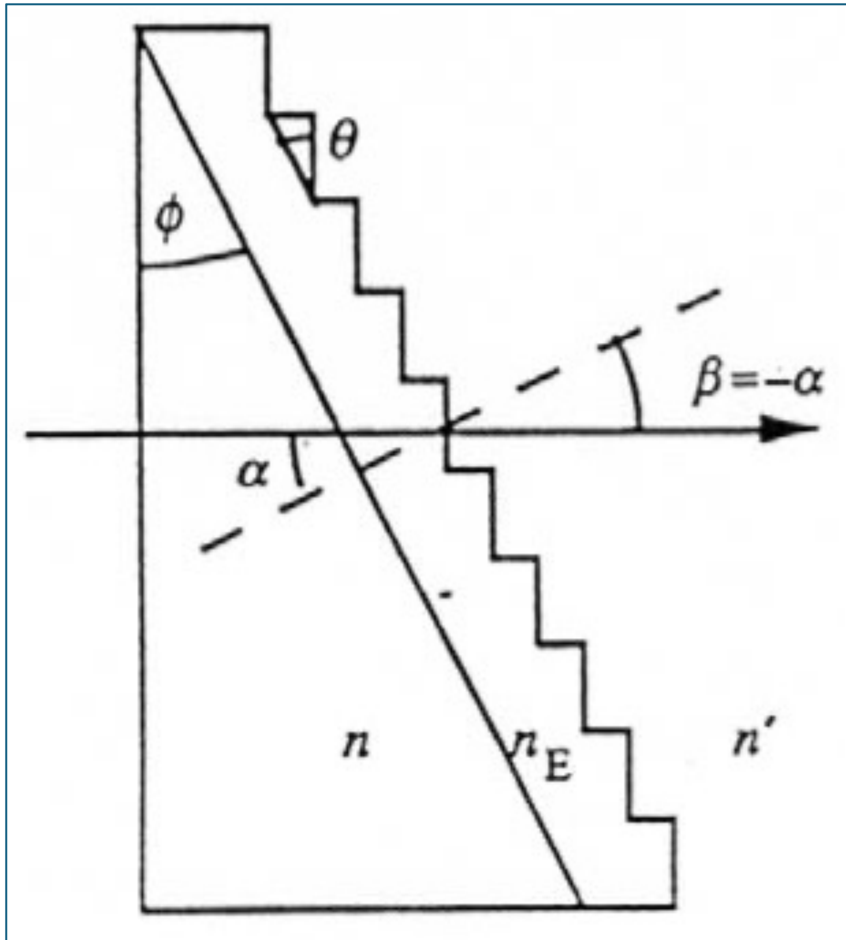
5.2.4 Grisms

- Grisms
  - hypotenuse(直角三角形の斜辺) > deviation of the prism
  - to bring the first order of diffraction on axis
- it can be placed in a filter wheel and treated like another filter.

$$m\lambda_c T = (n-1)\sin\phi$$

$$R = \frac{EFL}{2d_{pix}}(n-1)\tan\phi$$

- T = 1/d
- λ_c : central wavelength
- EFL : the effective focal length of the camera system
- R ~ 500-2000



Reference

https://science.uct.ac.za/sites/default/files/content_migration/science_uct_ac_za/1471/files/spec1.pdf

https://spectroscopy.wordpress.com/2020/05/12/basics-on-prisms-and-diffraction-gratings-part-1/