AGNAGN Seminar Sec.4-4.2

Tomoya Yukino

4 Calculation of Emitted Spectrum

4.1 Introduction

Radiation from nebular: depends on

1. Abundance
2. Temperature
3. Ionization
4. Density…

Emission line from gas nebular

$$\begin{cases} \textit{Collisionally excited line} \begin{cases} \textit{Forbidden line}(Opt, IR) \\ \textit{Permitted line}(UV) \end{cases} \\ \textit{Recombination line}(HI, HeI, HeII) \end{cases}$$

⇐Radiative transitions are forbidden by the parity selection rule

Strength of emission line: possible to be calculated from
①Cooling rate, ②thermal equilibrium

4.2 Optical Recombination Lines

HI recombination line: emitted by H atoms

(capture electron into excited level

⇒downward radiative transition)

Very low density(Case A):

Consider only captures and downward radiative transition

⇒equation of statistical equilibrium(level nL):

$$n_p n_e \alpha_{nL}(T) + \sum_{n'>n}^{\infty} \sum_{L'} n_{n'L'} A_{n'L',nL} = n_{nL} \sum_{n''=1}^{n-1} \sum_{L''} A_{nL,n''L''}. \quad (4.1)$$

Population in the level nL(thermodynamic equilibrium):

Saha equation
+
Boltzman equation

$$n_{nL} = (2L+1)\left(\frac{h^2}{2\pi mkT}\right)^{3/2} \exp(X_n/kT) n_p n_e, \quad (4.4)$$

Population in the level nL(general)

$$n_{nL} = b_{nL}(2L+1)\left(\frac{h^2}{2\pi mkT}\right)^{3/2} \exp(X_n/kT) n_p n_e, \qquad (4.6)$$

(4.1)+(4.6)⇒(4.7)
(b_{nL}:dimensionless factors,
$b_{nL} = 1$ in thermodynamic equilibrium)

$$\begin{aligned} &\alpha_{nL} \frac{1}{(2L+1)} \left(\frac{2\pi mkT}{h^2}\right)^{3/2} \exp(-X_n/kT) \\ &\quad + \sum_{n'>n}^{\infty} \sum_{L''} b_{n'L'} A_{n'L',nL} \left(\frac{2L'+1}{2L+1}\right) \exp\left[(X_{n'} - X_n)/kT\right] \qquad (4.7) \\ &= b_{nL} \sum_{n''=1}^{n-1} \sum_{L''} A_{nL,n''L''}, \end{aligned}$$

$b_{nL}(n > n_k)$ are known⇒All b_{nL} can be derived with (4.7)
(each equation contain a single unknown b_{nL})

Cascade matrix $\boldsymbol{C(nL, n'L')}$:

The probability that population of nL is followed by a transition to n'L' via **ALL** possible cascade routes

Probability matrix P(nL, n’L’):

The probability that population of nL is followed by a transition to n’L’ by a **DIRECT RADIATIVE TRANSITION**

$$C_{nL,n'L'} = \sum_{n''>n'}^{n} \sum_{L''=L'\pm 1} C_{nL,n''L''} P_{n''L'',n'L'}. \tag{4.10}$$

$$C_{nL,nL''} = \delta_{LL''}, \tag{4.9}$$

C(nL n’L’) is convenient tp express the solutions of (4.1):

$$n_p n_e \sum_{n'=n}^{\infty} \sum_{L'=0}^{n'-1} \alpha_{n'L'}(T)\, C_{n'L',nL} = n_{nL} \sum_{n''=1}^{n-1} \sum_{L''=L\pm 1} A_{nL,n''L'}. \tag{4.11}$$

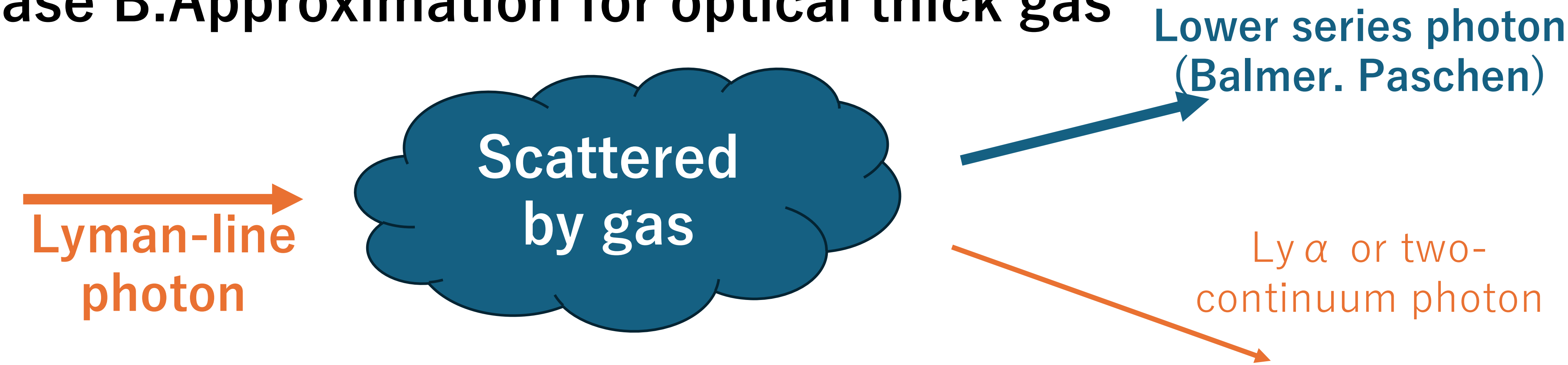
$$n_p n_e \sum_{n'=n}^{\infty} \sum_{L'=0}^{n'-1} \alpha_{n'L'}(T)\, C_{n'L',nL} = n_{nL} \sum_{n''=1}^{n-1} \sum_{L''=L\pm1} A_{nL,n''L'}. \quad (4.11)$$

Derive $C_{nL,n'L'}$, $\alpha_{n'L'}$ and extrapolate these series as $n \to \infty$
⇒Find n_{nL} with (4.11)
⇒Calculate emission coefficient $j_{nn'}$

$$j_{nn'} = \frac{h\nu_{nn'}}{4\pi} \sum_{L=0}^{n-1} \sum_{L'=L\pm1} n_{nL} A_{nL,n'L'}. \quad (4.12)$$

Observation for Case A nebular:
Optically thin, small amount of gas
⇒**too faint, hard to be observed**

Case B:Approximation for optical thick gas



Photon emitted in an $n^2P - 1^2S$ transition:
Absorbed immediately and raise another atom to n^2P level
⇒**Terminal of equilibrium equation: n’’=1(Case A)⇒2(Case B)**

$$n_p n_e \sum_{n'=n}^{\infty} \sum_{L'=0}^{n'-1} \alpha_{n'L'}(T)\, C_{n'L',nL} = n_{nL} \sum_{n''=2}^{n-1} \sum_{L''=L\pm1} A_{nL,n''L''}. \qquad (4.11)$$

Table 4.1

H I recombination lines (Case A, low-density limit)

	T			
	2,500 K	5,000 K	10,000 K	20,000 K
$4\pi j_{H\beta}/n_e n_p$ (erg cm^3 s^{-1})	2.70×10^{-25}	1.54×10^{-25}	8.30×10^{-26}	4.21×10^{-26}
$\alpha^{eff}_{H\beta}$ (cm^3 s^{-1})	6.61×10^{-14}	3.78×10^{-14}	2.04×10^{-14}	1.03×10^{-14}
Balmer-line intensities relative to Hβ				
$j_{H\alpha}/j_{H\beta}$	3.42	3.10	2.86	2.6
$j_{H\gamma}/j_{H\beta}$	0.439	0.458	0.470	0.4
$j_{H\delta}/j_{H\beta}$	0.237	0.250	0.262	0.2
$j_{H\epsilon}/j_{H\beta}$	0.143	0.153	0.159	0.1

Case A

Case B

Table 4.2

H I recombination lines (Case B, low-density limit)

	T			
	2,500 K	5,000 K	10,000 K	20,000 K
$4\pi j_{H\beta}/n_e n_p$ (erg cm^3 s^{-1})	3.72×10^{-25}	2.20×10^{-25}	1.24×10^{-25}	$6.62 \times 10^{-}$
$\alpha^{eff}_{H\beta}$ (cm^3 s^{-1})	9.07×10^{-14}	5.37×10^{-14}	3.03×10^{-14}	1.62×10^{-1}
Balmer-line intensities relative to Hβ				
$j_{H\alpha}/j_{H\beta}$	3.30	3.05	2.87	2.76
$j_{H\gamma}/j_{H\beta}$	0.444	0.451	0.466	0.474
$j_{H\delta}/j_{H\beta}$	0.241	0.249	0.256	0.262
$j_{H\epsilon}/j_{H\beta}$	0.147	0.153	0.158	0.162
$j_{H8}/j_{H\beta}$	0.0975	0.101	0.105	0.107
$j_{H9}/j_{H\beta}$	0.0679	0.0706	0.0730	0.0744
$j_{H10}/j_{H\beta}$	0.0491	0.0512	0.0529	0.0538

Hydrogen-like ion(like HeII)

Z: nuclear charge

Transition probabilities: **proportional to Z^4**

Recombination coefficient: $\alpha_{nL}(Z,T) = Z\alpha_{nL}(1, T/Z^2)$

⇒**Possible to derive emission coefficient:**

$$j_{nn'}(Z,T) = Z^3 j_{nn'}(1, T/Z^2)$$

Table 4.3

He II recombination lines (Case B, low-density limit)

	T			
	5,000 K	10,000 K	20,000 K	40,000 K
$4\pi j_{\lambda 4686}/n_e n_{He^{++}}$ (erg cm^3 s^{-1})	3.14×10^{-24}	1.58×10^{-24}	7.54×10^{-25}	3.48×10^{-25}
$\alpha^{eff}_{H\beta}$ (cm^3 s^{-1})	7.40×10^{-13}	3.72×10^{-13}	1.77×10^{-13}	8.20×10^{-14}
"Balmer"-line ($n \to 2$) intensities relative to λ4686				
$j_{32}/j_{\lambda 4686}$	0.560	0.625	0.714	8.15
$j_{42}/j_{\lambda 4686}$	0.154	0.189	0.234	2.84
$j_{52}/j_{\lambda 4686}$	0.066	0.084	0.106	1.32

Effect of collisional transition

①HI, HeII recombination line

1. nL→n(L±1) transition (Largest collisional cross sections)

Collisions with protons is more effective for angular momentum changing transitions

Equilibrium equations(include collisional transitions):

$$n_p n_e \alpha_{nL}(T) + \sum_{n'>n}^{\infty} \sum_{L'=L\pm1} n_{n'L'} A_{n'L',nL} + \sum_{L'=L\pm1} n_{n'L'} n_p q_{nL',nL} = n_{nL} \left[\sum_{n''=n_0}^{n-1} \sum_{L''=L\pm1} A_{nL,n''L''} + \sum_{L''=L\pm1} n_p q_{nL,nL''} \right] \quad (4.16)$$

$$q_{nL,n'L'} \equiv q_{nL,n'L'}(T) = \int_0^\infty u\sigma(nL \rightarrow n'L') f(u)\, du \ [\mathrm{cm}^3\ \mathrm{s}^{-1}] \quad (4.17)$$

$$n_{nL} = \frac{(2L+1)}{n^2} n_n, \quad (4.18)$$

※In He^{++} zone, both H^+ and He^{++} must be considered

2. nL→(n±1)(L+x) transition (2nd largest collisional cross sections)

- **x=±1 transition is the strongest**
- **It can be included in equilibrium equations by a straightforward generalization**

We can consider all collisions by the same approach:

- **More independent to n_e**
- **Enable the H-line emission coeficient**

Table 4.4
H I recombination lines (Case B)

	T								
	5,000 K			10,000 K			20,000 K		
n_e (cm^{-3})	10^2	10^4	10^6	10^2	10^4	10^6	10^2	10^4	10^6
$4\pi j_{H\beta}/n_e n_p$ (10^{-25} erg cm^3 s^{-1})	2.20	2.22	2.29	1.23	1.24	1.25	0.658	0.659	0.661
$\alpha^{eff}_{H\beta}$ (10^{-14} cm^3 s^{-1})	5.37	5.43	5.59	3.02	3.03	3.07	1.61	1.61	1.62
Balmer-line intensities relative to Hβ									
$j_{H\alpha}/j_{H\beta}$	3.041	3.001	2.918	2.863	2.847	2.806	2.747	2.739	2.725

②HeI recombination line

All transition probabilities between singlet and triplet are small

⇒**Possible to be treated them as separate systems**

1. Triplet

   Transitions to 1^1S level do not occur

   ⇒**Follow Case B**
2. Singlet

   Case B is ordinally a better approximation

HeI $1^{\mathbf{1}}\boldsymbol{S}-\boldsymbol{n}^{\mathbf{1}}\boldsymbol{P}$ line photons can photoionize $H^{\mathbf{0}}$

⇒Destroyed before they are converted into lower-energy photons

Table 4.6

He I recombination lines (Case B)

	T						
	5,000 K		10,000 K			20,000 K	
n_e (cm^{-3})	10^2	10^4	10^2	10^4	10^6	10^2	10^4
$4\pi j_{\lambda 4471}/n_e n_{He^+}$ (10^{-25} erg cm^3 s^{-1})	1.15	1.18	0.612	0.647	0.681	0.301	0.408
α^{eff}_{4471} (10^{-14} cm^3 s^{-1})	2.60	2.67	1.39	1.47	1.54	0.683	0.925
Triplet lines relative to $\lambda 4471$							
$j_{\lambda 5876}/j_{\lambda 4471}$	2.93	2.92	2.67	2.90	2.97	2.62	3.62
$j_{\lambda 4026}/j_{\lambda 4471}$	0.460	0.461	0.476	0.469	0.467	0.484	0.437