Section6 ex4&10

Misato Fujii

4 Given the relation shown below for the angular blur size diameter (radians) due to spherical aberration in a single lens of refractive index $n = 1.5$ and focal ratio $f/2$, determine the linear diameter of the image of a point source if the focal length of the camera is 50 mm:

$$\beta = \frac{n(4n-1)}{128(n+2)(n-1)^2} \frac{1}{(f/\text{number})^3}$$

How does this compare with the size of a typical CCD pixel? How can this limitation be overcome in practice?

Refractive index $n = 1.5$

Focal ratio $f/2$

Focal length of the camera 50mm

Angular blur size diameter $\beta = \frac{n(4n-1)}{128(n+2)(n-1)^2} \frac{1}{(f/number)^3} = \frac{1.5\times 5}{128\times 3.5\times 0.25} \frac{1}{2^3} = 0.00837(\text{radians})$

Linear diameter $\beta f = 0.00837(\text{rad})\times 50(\text{mm}) = 0.419(\text{mm}) = 419(\text{um}) \gg$ size of a typical CCD pixel

To overcome the limitation

→ arrange many small lenses to make a 419um lens or arrange typical CCD pixels

10 Derive expressions for the étendue of a spectrometer with an entrance aperture defined by the slit, and derive a similar expression for a seeing-limited camera. See theory below.

étendue : luminosité or throughput
Surface element of area(dS), brightness(B) radiates into solid angle($d\Omega$)
→flux(energy rate, J/s) $dF = B\cos\theta dS d\Omega$
dS:entrance pupil, seeing disk (Airy diffraction disk), or entrance slit of spectrometer
$d\Omega = 2\pi \sin\theta d\theta$
→$F = 2\pi B dS \int_0^{\theta_m} \cos\theta \sin\theta \, d\theta = BU$
θ_m:maximum value of the half-angle of the cone of rays from dS
$U = \pi \sin^2\theta_m dS$: *étendue*

Evaluate *étendue* for different systems
→ $U = A\Omega$
A:area of entrance aperture, Ω:solid angle subtended by the entrance pupil
No loss by absorption or reflection
→$U = A\Omega$ is conversed

10 Derive expressions for the étendue of a spectrometer with an entrance aperture defined by the slit, and derive a similar expression for a seeing-limited camera. See theory below.

$$A = \frac{\pi}{4} D_{coll}^2$$

・spectrometer　(w: slit width, h:slit height)

$$\Omega = \frac{wh}{f_{coll}^2}$$

$$U = A\Omega = \frac{\pi}{4} D_{coll}^2 \frac{wh}{f_{coll}^2}$$

・seeing-limited camera (θ:seeing limited diameter)

$$\Omega = \frac{\pi\theta^2}{4f_{coll}^2}$$

$$U = A\Omega = \frac{\pi^2}{16} D_{coll}^2 \frac{\theta^2}{f_{coll}^2}$$