McLeanゼミ

Sec9.7-End of Sec9

小島裕樹

Contents

9.7 Spectroscopy

- Spectroscopic applications put the most stringent requirements on detectors to achieve the lowest possible noise performance.
- Diffraction gratings are mainly used to disperse light in large astronomical spectrographs, although prisms with transmission gratings are also frequently used.

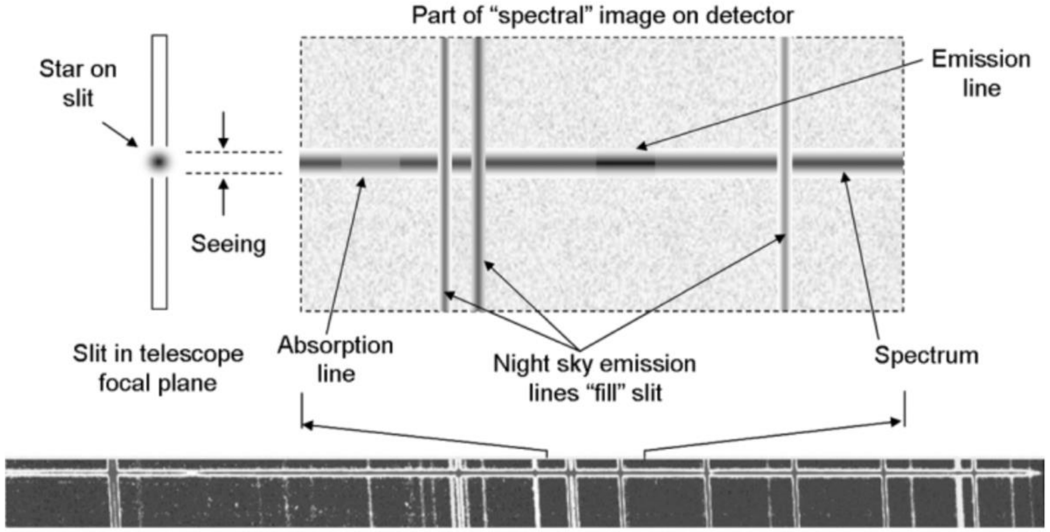


Figure 9.14. An illustration of the typical appearance of a spectrum on an array detector, including the presence of night-sky lines. The width is determined by the seeing.

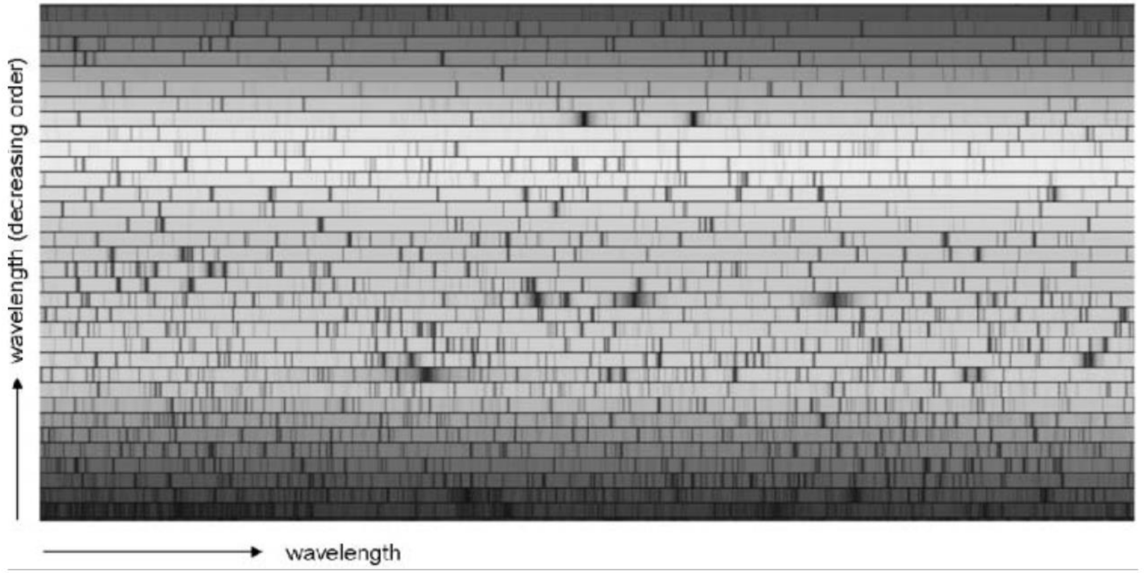


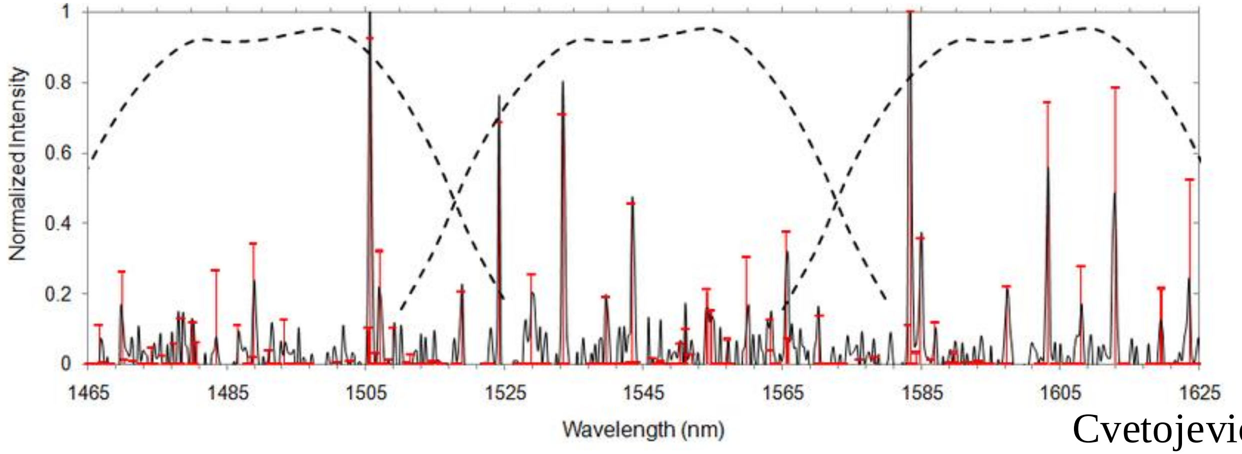
Figure 9.15. A cross-dispersed echelle spectrometer fills the detector array with many spectral segments.

9.7 Spectroscopy

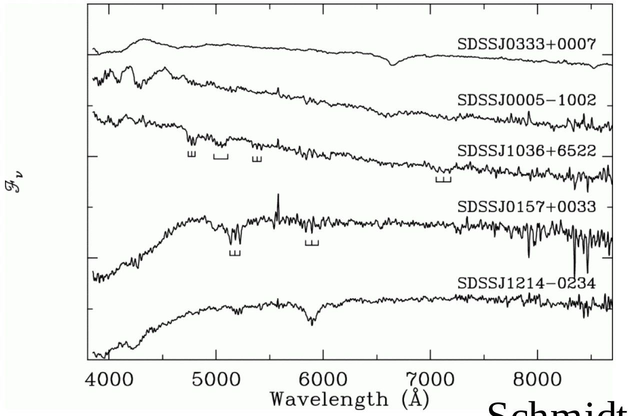
- Spectroscopic calibrations proceed in much the same way as with imaging.
- A flat-field is required to remove optical interference effects caused by the near-monochromatic light and variations in the thickness of thinned backside-illuminated CCDs.
- Bias frames must precede the derivation of the flat-field, but now, due to the much weaker signals and longer integrations, dark current may be more dominant.
- Many dark frames may need to be averaged and subtracted from the object frame.

9.7 Spectroscopy

- Wavelength calibration deciding the relationship between pixel number and wavelength is done by using arc lamp (emission-line) spectra that contains numerous lines with accurately known wavelengths.
- In the NIR, using the numerous, sharp OH night-sky lines is convenient.
- Atmospheric extinction corrections and correction to absolute flux levels are accomplished by comparing the observed spectrum to that of a flux standard, such as A0 V stars not located far away and therefore not reddened by dust and white dwarfs with almost featureless spectra.



Cvetojevic et al. (2009)



Schmidt et al. (2003)

9.7 Spectroscopy

- A summary of the key steps is as follows:

(1) Identify the direction of dispersion (is increasing wavelength the same as increasing pixel numbers?)

(2) Interpolate over dead pixels or columns because these outliers may ruin the subsequent steps but record these locations to remember there was no real data in those pixels.

(3) Sum up and normalize to unity the flat-fields. Flat-fields are sometimes taken with the spectrograph slit wide open if that is an option. In this way, the orders overlap considerably giving a uniform illumination on the CCD when viewing a quartz lamp illuminating a white screen on the inside of the dome.

(4) Divide the observed stellar spectra by the flat-field to remove pixel-to-pixel sensitivity variations.

9.7 Spectroscopy

(5) Some software packages like IRAF require that you observe a bright star to some extent in order to define the positions of the orders across the CCD. The program (e.g., “ptrace”) will then know where to find spectra.

(6) Extract the rectangular subsets of CCD pixels corresponding to the stellar spectrum. Do the same for the arc lamp using the normal slit width and quartz lamp exposures. In IRAF this is done with a program called “apsum”.

(7) Divide the flat-fielded stellar spectra by the “white-light” spectrum obtained with the normal slit and quartz lamp. This white light spectrum must itself be flat-fielded with the open-slit flat-field before this division. The purpose of this step is to remove interference fringe effects rather than pixel-to-pixel variations in sensitivity.

(8) Identify emission lines in the arc lamp used as a wavelength calibrator. The most commonly used lamp is a thorium-argon lamp spectrum. This is a painstaking step, but an important one if accurate transformations from pixels to wavelengths are required.

9.8 Polarimetry

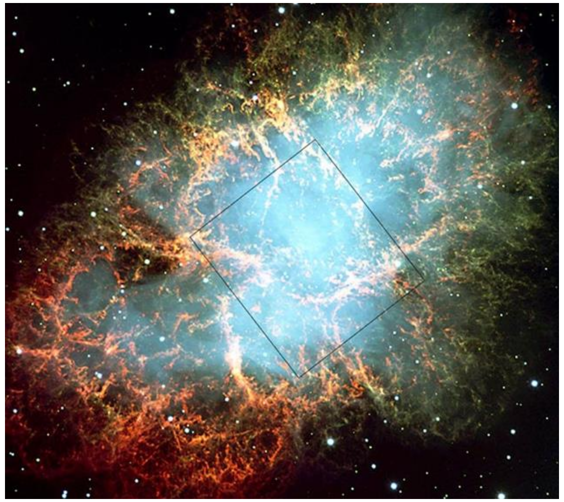
- Polarimeter rotates in the beam, so dust spots and other artifacts cannot be flat-fielded with just one waveplate orientation.

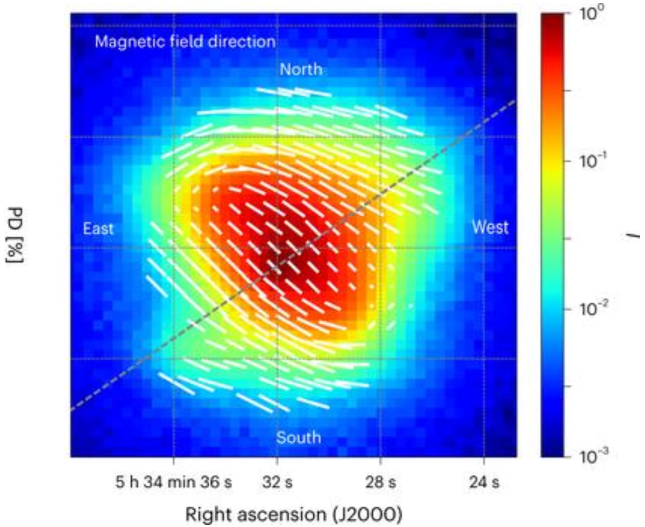
→A set of flat-fields corresponding to different positions of the polarimeter's waveplate are needed.

- If a half-waveplate is used to determine the linear polarization components $Q/I = p \cos 2\Theta$ and $U/I = p \sin 2\Theta$, then Q/I and U/I can be measured by the difference in counts at four waveplate rotations from an arbitrary starting point.
- Here, I is the total intensity, p is the percentage or fractional linear polarization, and Θ is the direction of vibration of the electric vector of the linearly polarized component.

9.8 Polarimetry

- Stars or objects of known polarization like the Crab Nebula should be observed to verify the efficiency of the measurement and determine any scale factor.
- By observing completely unpolarized sources we can discover the "instrumental polarization" inherent in the system. The instrumental values of Q and U should be subtracted before p and Θ are calculated.


Credit:European Southern Observatory (ESO)



Bucciantini et al. (2023)

9.9 Signal-to-noise calculations

- The simplest approach to analyzing a CCD image is to construct a “final frame” by subtracting dark current and normalize with a flat-field.

$$\text{FINAL FRAME} = \frac{\text{OBJECT FRAME} - \text{DARK FRAME}}{\text{FLAT FRAME} - \text{DARK FRAME}}$$

- For infrared arrays, the night sky is often used as the flat-field source, so SKY FRAME can be used instead of FLAT FRAME in the equation.
- Here, these noise sources are taken into account:

  (1) readout noise, *R* electrons

  (2) photon (Poisson) noise on the signal (*S*) from the object

  (3) photon (Poisson) noise on the signal (*B*) from the sky background

  (4) shot noise on the dark-current signal (*D*)

9.9 Signal-to-noise calculations

- The number of OBJECT, SKY/FLAT, and DARK frames combined or "co-added" to form the FINAL FRAME are not necessarily equal.
- But for simplicity, we assume that the normal practice pertains of keeping the same exposure time (*t*) for each image.
- Suppose we have n_O OBJECT FRAMES, n_B SKY BACKGROUND or FLAT FRAMES, and n_D DARK FRAMES and let

$T = tn_O$ be the TOTAL integration time accumulated on the OBJECT FRAMES;

$f =$ the ratio of the source signal to that of the "background" signal per pixel;

$\varepsilon_B = n_O/n_B$ is the ratio of the number of object frames to background frames;

$\varepsilon_D = n_O/n_D$ is the ratio of the number of object to dark-current frames;

9.9 Signal-to-noise calculations

- Then, when a source is covering *n* pixels on the CCD, we can estimate a total signal-to-noise ratio (S/N) of

$$\frac{S}{N} = S\sqrt{T}\left[u_r^2 + S + \sum_{i=1}^{n}\left\{\begin{array}{c}\left(B + D + \frac{R^2}{t}\right) + \varepsilon_D\left(D + \frac{R^2}{t}\right) + \\ + (1+f)^2\varepsilon_B\left(B + D + \frac{R^2}{t}\right) + (1+f)^2\varepsilon_D\left(D + \frac{R^2}{t}\right)\end{array}\right\}\right]^{-1/2}$$

- S : the total object signal summed over n pixels (i.e., $S = \Sigma\, (S_i)$)
- u_r : the average, over *n* pixels, of any residual error due to failure in the flat-field
- *B*, *D*, and *f* are different from pixel to pixel.

9.9 Signal-to-noise calculations

- The terms in the denominator of S/N (↓) can be understood as follows.

$$\sqrt{\overset{(1)}{u_r^2} + \overset{(2)}{S} + \sum_{i=1}^{n}\left\{\overset{(3)}{\left(B + D + \frac{R^2}{t}\right)} + \overset{(4)}{\epsilon_D\left(D + \frac{R^2}{t}\right)} + \overset{(5)}{(1+f)^2\epsilon_B\left(B + D + \frac{R^2}{t}\right)} + \overset{(6)}{(1+f)^2\epsilon_D\left(D + \frac{R^2}{t}\right)}\right\}}$$

- (1) is due to the residual non-uniformity (u_r) .
- (2) is the Poisson noise in the source signal itself.
- (3) is the background and dark-current Poisson noise in quadrature with the readout noise in the raw source frame.
- (4) is the error due to subtracting a dark frame from the object frame. The more dark frames, the smaller ε_D and the less significant this term.

9.9 Signal-to-noise calculations

$$\sqrt{\overset{(1)}{u_r^2} + \overset{(2)}{S} + \sum_{i=1}^{n}\left\{\overset{(3)}{\left(B + D + \frac{R^2}{t}\right)} + \overset{(4)}{\epsilon_D\left(D + \frac{R^2}{t}\right)} + \overset{(5)}{(1+f)^2\epsilon_B\left(B + D + \frac{R^2}{t}\right)} + \overset{(6)}{(1+f)^2\epsilon_D\left(D + \frac{R^2}{t}\right)}\right\}}$$

- (5) comes from the application of the SKY/FLAT correction and (6) is the result of dark subtraction for the SKY/FLAT term. The more SKY/FLATS that are used the smaller ε_B.
- The additional scaling factor of $(1+f)^2$ arises when the object frame is divided by the flat-field and the ratio renormalized by multiplying by the mean of the flat-field.

9.9 Signal-to-noise calculations

- Suppose you observe a point-like object that has a seeing disk of diameter θ_{FWHM} [arcsec], corresponding to the full width at half maximum intensity (FWHM), with a CCD whose pixel has a side length θ_{pix} [arcsec] , then the number of pixels covered by the star's image is approximately

$$n_{\mathrm{pix}} = \frac{\pi}{4}\left(\frac{\theta_{\mathrm{FWHM}}}{\theta_{\mathrm{pix}}}\right)^2$$

and the summations must be taken over n_{pix} pixels to estimate the S/N.

- If the object is extended and its angular size is much larger than θ_{FWHM} , then it is more convenient to deal with “surface brightness” in magnitudes per square arcsecond, and so $n_{\mathrm{pix}} = 1$ and each pixel is treated separately.

9.9 Signal-to-noise calculations

- In the ideal case with accurate calibration data so that ε_B and ε_D are very small, and if u_r , the residual non-flatness, is negligible, then S/N is

$$\frac{S}{N} = \frac{S\sqrt{n_o t}}{\sqrt{S + n_{\text{pix}}\left(S + B + \frac{R^2}{t}\right)}}$$

typo for D?

- There are two further simplifying cases:

1. Background-limited or "sky-limited":
   $B >> (D + R^2/t)$ and $S << B$

$$\frac{S}{N} = S\sqrt{T}[(n_{\text{pix}}B)]^{-1/2}$$

2. Detector noise-limited :
   $R^2/t >> (B + D)$ and a weak signal S

$$\frac{S}{N} = \frac{S\sqrt{T}}{\left[n_{\text{pix}}\left(\frac{R^2}{t}\right)\right]^{1/2}} = \frac{St}{R}\sqrt{\frac{n_o}{n_{\text{pix}}}}$$

9.9 Signal-to-noise calculations

Background-limited example:

Suppose the total source (S) is only 1% of the brightness of the sky in a single pixel and the source is spread over 4 pixels. S/N = 1 is required to just barely detect this source. Therefore, BT = 40,000 photoelectrons. If the sky background (B) gives about 400 electrons/s per pixel, then this observation will take 100 s.

Detector noise-limited example:

In a high-resolution spectrograph the background plus dark current is 0.1 electrons/s/pixel, and so for t = 1,000 s we are formally readnoise-limited if $R^2/t >$ 0.1 or $R^2 >$ 100 (i.e., if R is larger than 10 electrons).

9.9 Signal-to-noise calculations

- The highest signal-to-noise ratios are obtained when
  - (a) sufficiently accurate calibration frames are obtained
  - (b) the readout noise and dark current are as small as possible
  - (c) the on-chip integration time is as long as possible
  - (d) quantum efficiency and telescope area are as large as possible.
- The estimation of the power [W] collected by a telescope of area A_{tel} [cm^2] in a wavelength interval of $\Delta\lambda$ [μm] from a source of apparent magnitude m (below the Earth's atmosphere), transmitted by an optical system of efficiency τ (<1) onto a CCD detector of quantum efficiency η (<1) is

$$P(\lambda) = \tau(\lambda)\eta(\lambda)A_{\text{tel}}\,\Delta\lambda\,F_\lambda(0) \times 10^{-0.4m} \quad \text{W}$$

where F_λ (0) is the flux [W cm^{-2} μm^{-1}] from a zeroth magnitude standard star above the atmosphere.

9.9 Signal-to-noise calculations

Table 9.4. Absolute flux from a zero-magnitude star like Vega.

Symbol	λ (μm)	ν (Hz)	F_λ (W cm^{-2} μm^{-1})	F_ν (Jy)*
U	0.36	8.3×10^{14}	4.35×10^{-12}	1,880
B	0.43	7.0×10^{14}	7.20×10^{-12}	4,440
V	0.54	5.6×10^{14}	3.92×10^{-12}	3,810
R	0.70	4.3×10^{14}	1.76×10^{-12}	2,880
I	0.80	3.7×10^{14}	1.20×10^{-12}	2,500
J	1.25	2.4×10^{14}	2.90×10^{-13}	1,520
H	1.65	1.8×10^{14}	1.08×10^{-13}	980
K	2.2	1.36×10^{14}	3.8×10^{-14}	620
L	3.5	8.6×10^{13}	6.9×10^{-15}	280
M	4.8	6.3×10^{13}	2.0×10^{-15}	153
N	9.1	3.0×10^{13}	1.09×10^{-16}	37

9.9 Signal-to-noise calculations

- The transmission factor τ is the product of all the transmission od reflectance factors in the system.
- For example, assume two telescope mirrors with a 95% reflectance each, six lenses with 96% transmission each, and a filter with 80% transmission, then the total transmission is

$$\tau = \tau_{\mathrm{tel}}\tau_{\mathrm{optics}}\tau_{\mathrm{filter}} = (0.95)^2(0.96)^6(0.8) = 0.57$$

- A single photon has energy hc/λ , so the photoelectron detection rate is

$$S(\lambda) = (hc)^{-1}\tau(\lambda)\eta(\lambda)A_{\mathrm{tel}}\lambda\,\Delta\lambda\,F_\lambda(0) \times 10^{-0.4m} \quad \text{electrons/s}$$

9.9 Signal-to-noise calculations

- Dividing S (λ) by g [e^-/DN] gives the observed signal rate in [DN/s], and if we set this rate at 1 DN/s then we can derive the corresponding magnitude m_{zp} which is the "zeropoint" of the instrument scale.

$$m_{zp} = 2.5 \log\left\{\frac{\tau\eta\lambda \, \Delta\lambda \, A_{tel} F_\lambda(0)}{hcg}\right\}$$

- We can also get m_{zp} by observing a standard star of known magnitude.
- Having obtained m_{zp} by observations we can then derive the product $\tau\eta$ which describes the system efficiency in the given passband:

$$2.5 \log(\tau\eta) = m_{zp} - 2.5 \log\left\{\frac{\lambda \, \Delta\lambda \, A_{tel} F_\lambda(0)}{hcg}\right\}$$

9.9 Signal-to-noise calculations

Example:

Suppose in the K'-band (λ = 2.125 μm, $\Delta\lambda$ = 0.35 μm) the zeropoint is observed to be 20.4 on a telescope with an area of 72,236 cm^2 using a camera with a gain of 25 electrons/DN. Taking $F_\lambda(0) = 4.34 \times 10^{-14}$ W cm^{-2} μm^{-1} gives

$$2.5\log(\tau\eta) = 20.4 - 2.5\log\left\{\frac{2.125 \times 0.35 \times 72{,}236 \times 4.34 \times 10^{-14}}{1.99 \times 10^{-19} \times 25}\right\} = -1.28$$

which corresponds to $\tau\eta$ =0.31 (or 31%).

9.9 Signal-to-noise calculations

- The signal from the sky or background (which includes thermal emission from the telescope at infrared wavelengths) depends on many things like temperature and the amount of moonlight.
- To quantify the background a simple approach is to use the same form as before.

$$B(\lambda) = (hc)^{-1}\tau(\lambda)\eta(\lambda)A_{\text{tel}}\lambda\,\Delta\lambda\,F_\lambda(0) \times 10^{-0.4m_{\text{sky}}}\theta^2_{\text{pix}} \quad \text{electrons/s}$$

- In the thermal infrared, F_λ can be replaced with the Planck function $B(\lambda)$ and an emissivity factor ε.

9.9 Signal-to-noise calculations

- B is proportional to $A_{\mathrm{tel}}\theta_{\mathrm{pix}}^2$ and

$$A_{\mathrm{tel}}\theta_{\mathrm{pix}}^2 = \frac{\pi}{4} D_{\mathrm{tel}}^2 \left(206{,}265 \frac{d_{\mathrm{pix}}}{D_{\mathrm{tel}}(f/\#)_{\mathrm{cam}}}\right)^2 \propto \frac{1}{(f/\#)_{\mathrm{cam}}^2}$$

where d_{pix} is the pixel size of the detector and $(f/\#)_{\mathrm{cam}}$ is the focal ratio of the camera system.

- Therefore, for a given camera system, the background is independent of telescope diameter D_{tel} and depends only on focal ratio $(f/\#)_{\mathrm{cam}}$.
- Many optical telescopes have focal ratios of about $f/8$ whereas infrared telescopes use $f/36$ or larger to reduce the background.

9.9 Signal-to-noise calculations

Example:

Consider a CCD camera with $\tau = 0.5$ and $\eta = 0.5$ at a wavelength of 0.54 μm in the middle of the V band with a filter passband of $\Delta\lambda = 0.1$ μm on a 4 m class telescope (i.e., a 4 m diameter primary mirror with a 1 m Cassegrain hole in its center). With the appropriate value of $F_\lambda(0) = 3.92 \times 10^{-12}$ W cm^{-2} μm^{-1} ,

$$S = 3.136 \times 10^{10} \times 10^{-0.4m} \quad \text{electrons/s}$$

so that for $m = 17$ we obtain 5,000 electrons/s whereas for the extremely faint sources at $m = 27$, we detect only 0.5 electrons/s.

9.9 Signal-to-noise calculations

- A useful expression for predicting the "limiting magnitude" of an array camera in the background-limited or sky-limited case is here.

$$m = m_{zp} - 2.5 \log\left\{\frac{1}{g}\frac{S}{N}\sqrt{\frac{n_{pix}B}{T}}\right\}$$

This equation is derived from

$$\frac{S}{N} = S\sqrt{T}[(n_{pix}B)]^{-1/2} \qquad m_{zp} = 2.5 \log\left\{\frac{\tau\eta\lambda\,\Delta\lambda\,A_{tel}F_{\lambda}(0)}{hcg}\right\}$$

$$S(\lambda) = (hc)^{-1}\tau(\lambda)\eta(\lambda)A_{tel}\lambda\,\Delta\lambda\,F_{\lambda}(0) \times 10^{-0.4m} \quad \text{electrons/s}$$

- If the "limit" corresponds to S/N = 1, the limiting magnitude is

$$m_{lim} = m_{zp} - 2.5 \log\left\{\frac{1}{g}\sqrt{\frac{n_{pix}B}{T}}\right\}$$

9.9 Signal-to-noise calculations

Example:

A camera with a gain of 10 electrons/DN and a zeropoint of $m_{zp} = 18$ forms a star image across 5 pixels with an average background of 200 electrons/ s/ pixel. What is the limiting magnitude for a 1-hour exposure?

As we are assuming the background-limited case, we are not concerned with the way the exposure has been broken up ($n_0 = 1$) ; all that matters is $T = t = 3{,}600$ s and S/N = 1:

$$m = 18 - 2.5 \log\{0.1 \times 1 \times \sqrt{(1{,}000/3{,}600)}\} = 18 + 3.2 = 21.2$$

In reality, S/N calculations are often more complex than presented here because star images are not sharp-edged, filter transmissions are not simply represented by a wavelength interval, and the stellar energy spectrum is not the same as that of Vega.

9.10 Summary

- You can routinely determine important parameters in a CCD, such as quantum efficiency, readout noise, uniformity (flat-field), linearity, and gain by following the steps from quantum detection of photons to evaluation of fluxes, magnitudes, and signal-to-noise ratios by processing data frames.
- Good flat-field correction enables us to detect too faint sources to be apparent in the raw image.
- Relative photometry to an accuracy of 1/1000 mag is possible.
- How to calculate signals and backgrounds for a CCD and a formula for the S/N often called the “CCD equation” are described.
- When all the steps are followed, CCDs and infrared detectors become very powerful in photometers and spectrographs.