AGNAGN Seminar
Sec2-2.3

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2. Photoionization Equilibrium
2.1 Introduction

・emission nebulae: photoionization of a diffuse gas cloud by UV photons from a hot “exciting” star or from a cluster of exciting stars

・**ionization equilibrium** in the nebula: the balance between photoionizations and recombinations of electrons with the ion

・hydrogen: most abundant element

⇨first approximation to the structure of nebula: pure H cloud surrounding by a single hot star

・ionization equilibrium:

$$n(\mathrm{H}^0)\int_{\nu_0}^{\infty}\frac{4\pi J_\nu}{h\nu}a_\nu(\mathrm{H}^0)\,d\nu = n(\mathrm{H}^0)\int_{\nu_0}^{\infty}\phi_\nu a_\nu(\mathrm{H}^0)\,d\nu = n(\mathrm{H}^0)\Gamma(\mathrm{H}^0) = n_e n_p \alpha(\mathrm{H}^0, T)\ [\mathrm{cm}^{-3}\ \mathrm{s}^{-1}]$$

・J_ν: mean intensity of radiation at the point (per unit area, unit time, unit solid angle, unit frequency interval)

・$\phi_\nu = \frac{4\pi J_\nu}{h\nu}$: number of incident photons (per unit area, unit time, unit frequency interval time)

・$a_\nu(\mathrm{H}^0)$: ionization cross section for H by photons with energy $h\nu$

・$\Gamma(\mathrm{H}^0) = \int_{\nu_0}^{\infty}\phi_\nu a_\nu(\mathrm{H}^0)\,d\nu$: number of photoionizations (per H atom, unit time)

・$n(\mathrm{H}^0)$, n_e, n_p: neutral atom, electron, proton densities (per unit volume)

・$\alpha(\mathrm{H}^0, T)$: recombination coefficient

→right-hand side: number of recombinations (per unit volume, unit time)

2.1 Introduction

・to a first approximation, J_ν is the radiation emitted by the star reduced by the inverse-square effect of geometrical dilution

- $4\pi J_\nu = \frac{R^2}{r^2}\pi F_\nu(0) = \frac{L_\nu}{4\pi r^2}$ $[\mathrm{erg\,cm^{-2}\,s^{-1}\,Hz^{-1}}]$
  - R: radius of the star
  - $\pi F_\nu(0)$: flux of the surface of the star
  - r: distance from the star to the point
  - L_ν: luminosity of the star per unit frequency interval

2.1 Introduction

・at a typical point in a nebula, H is almost completely ionized because the UV field is so intense

- ・example: point in HII region
  - ・density 10H atoms and ions $/\mathrm{cm}^3$ $(n(\mathrm{H}) = 10\ \mathrm{cm}^{-3})$
  - ・5pc from a central O7.5 star $(T_* = 39{,}700K)$
  - ・$\mathrm{Q(H^0)} = \int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu} d\nu \approx 1 \times 10^{49}$ [photons s^{-1}]
  - ・$a_\nu(\mathrm{H^0}) \approx 6 \times 10^{-18}$ [cm^2]

    $\rightarrow \int_{\nu_0}^{\infty} \frac{4\pi J_\nu}{h\nu} a_\nu(\mathrm{H^0})\, d\nu \approx 1 \times 10^{-8} = \tau_{ph}^{-1}$ [s^{-1}]
    - ・τ_{ph}: lifetime of the atom before photoionization
  - ・$\alpha(\mathrm{H^0}, T) \approx 4 \times 10^{-13}$ [$\mathrm{cm^3 s^{-1}}$]
  - ・ξ: fraction of neutral H

    $\rightarrow n_e = n_p = (1 - \xi) n(\mathrm{H})$, $n(\mathrm{H^0}) = \xi n(\mathrm{H})$

  $\rightarrow \xi \approx 4 \times 10^{-4}$: nearly completely ionized

2.1 Introduction

・finite source of UV photons can't ionize an infinite volume

⇨if the star is in a sufficiently large gas cloud, there must be an outer edge to the ionized material

・thickness of transition zone between ionized and neutral gas is approximately one mean free path of an ionizing photon ($l \approx [n(\mathrm{H}^0)a_\nu]^{-1}\ \mathrm{cm}$)

・due to absorption

・same parameter before and $\xi \approx 0.5$

⇨thickness of transition zone: $d \approx \frac{1}{[n(\mathrm{H}^0)a_\nu]} \approx 0.1\mathrm{pc}$ or much smaller than the radius of the ionized nebula

⇨nearly completely ionized "Strömgren sphere" or HII region

・separated by a thin transition region from outer neutral gas cloud or HI region

2.1 Introduction

・In this chapter

・examine the photoionization cross section and recombination coefficient for H

→calculate the structure of hypothetical pure H regions

・consider the photoionization cross section and recombination coefficient for He (second most abundant element)

→calculate more realistic models of HII regions

・extend analysis to less abundant heavy elements

・do not strongly affect the ionization structure of the nebula, but are very important in thermal balance

2.2 Photoionization and Recombination of Hydrogen

- energy-level diagram of H
  - n: principal quantum number
  - L: angular momentum quantum number
    - S, P, D, F, ⋯→L=0, 1, 2, 3, ⋯
  - permitted transitions to levels n<4 (solid line)
    - for one-electron system, selection rule: $\Delta L = \pm 1$
  - mean lifetimes of the excited level:
    - $\tau_{nL} = \frac{1}{\sum_{n'<n}\sum_{L'=L\pm1} A_{nL,n'L'}}$ $(10^{-4} - 10^{-8}\ s)$
    - $A(nL, n'L')$: transition probabilities $(10^{4} - 10^{8}\ s^{-1})$

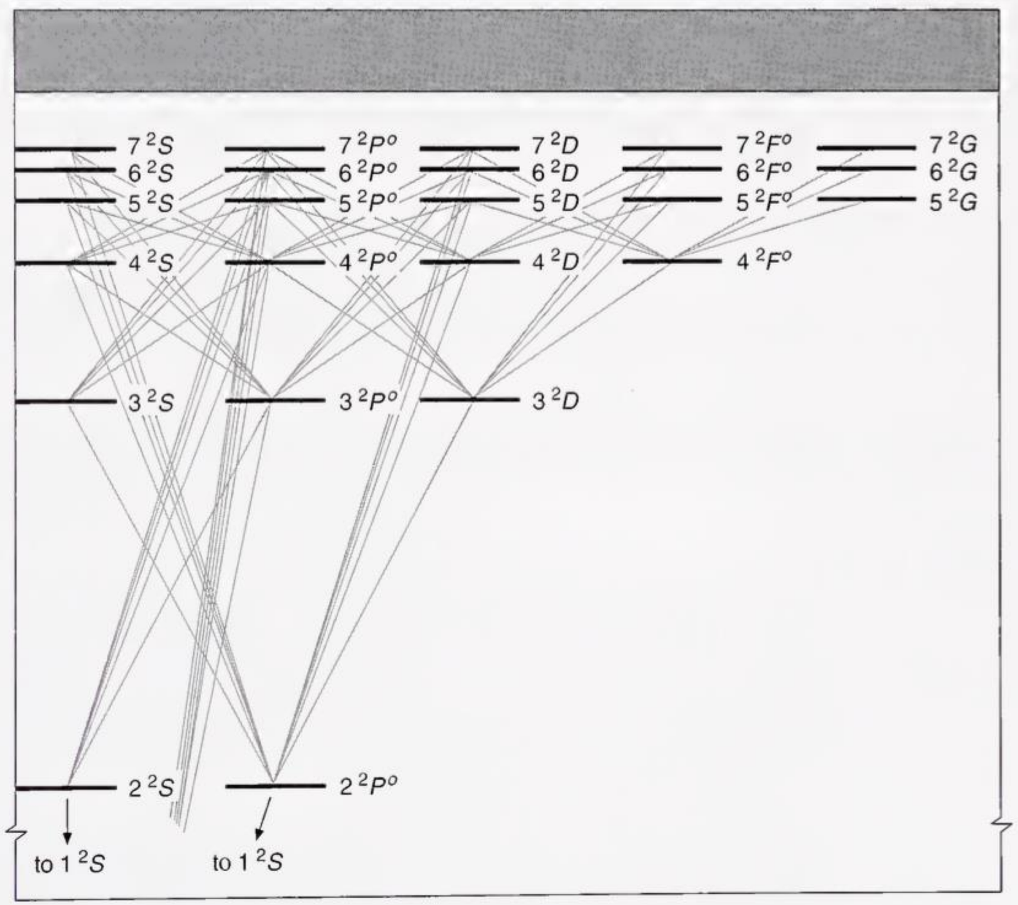

Figure 2.1
Partial energy-level diagram of H I, limited to $n \leq 7$ and $L \leq G$. Permitted radiative transitions to levels $n < 4$ are indicated by solid lines.

2.2 Photoionization and Recombination o

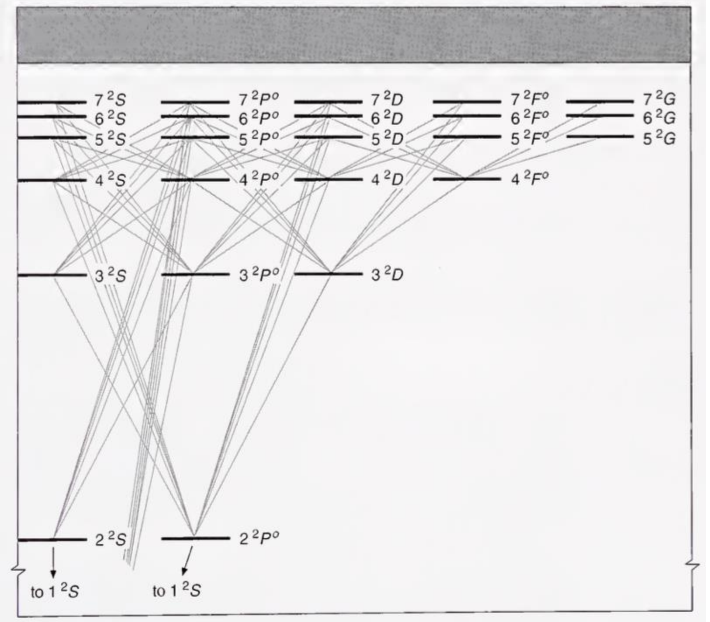

Figure 2.1
Partial energy-level diagram of H I, limited to $n \leq 7$ and $L \leq G$. Permitted radiative transitions to levels $n < 4$ are indicated by solid lines.

- exception: 2^2S level
  - not allowed one-photon downward transition
  - $2^2S \rightarrow 1^2S$ occur with emission of two photons
    - $A(2^2S, 1^2S) = 8.23\ \mathrm{s}^{-1}$, $\tau_{2^2S} = 0.12\ \mathrm{s}$

    →quite shorter than the mean lifetime of H atom before photoionization
      - $\tau_{ph} \approx 10^8\ \mathrm{s}$ (1^2S level), same order of magnitude for excited level

⇨we can consider that nearly all the H^0 is in 1^2S level

- photoionization from 1^2S is balanced by recombination to all levels
- recombination to excited level is quickly followed by radiative transitions downward to ground level

⇨simplify calculations of physical conditions in gaseous nebulae

2.2 Photoionization and Recombination of Hydrogen

・photoionization cross section for the 1^2S level of H^0 (or hydrogenic ion with nuclear charge Z):

・$a_\nu(Z) = \frac{A_0}{Z^2}\left(\frac{\nu_1}{\nu}\right)^4 \frac{\exp\{4-[(4\tan^{-1}\varepsilon)/\varepsilon]\}}{1-\exp(-2\pi/\varepsilon)}$ $[\mathrm{cm}^2]$ $(\nu \geq \nu_1)$

・$A_0 = \frac{2^9\pi}{3e^4}\left(\frac{1}{137.0}\right)\pi a_0^2 = 6.30 \times 10^{-18}$ $[\mathrm{cm}^2]$

・$\varepsilon = \sqrt{\frac{\nu}{\nu_1} - 1}$

・$h\nu_1 = Z^2 h\nu_0 = 13.6 Z^2$ eV (threshold energy)

→Figure2.2

・$a_\nu(Z)$ drops off rapidly with energy as ν^{-3} not too far above the threshold

・threshold for H: $\nu_0 = 3.29 \times 10^{15}\ s^{-1}$ or $\lambda_0 = 912\text{Å}$

⇨higher energy photons penetrate further into neutral gas before absorbed

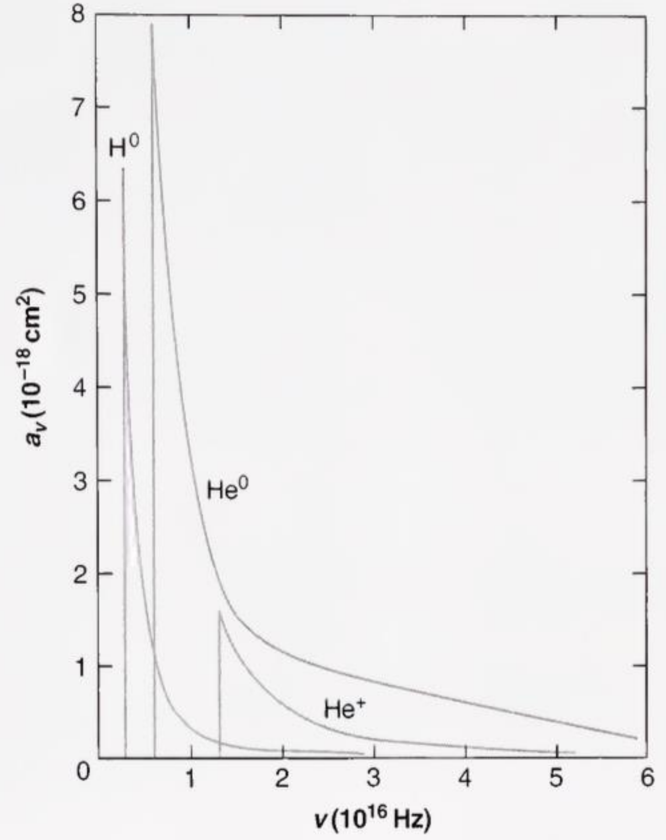

Figure 2.2

Photoionization absorption cross sections of H^0, He^0, and He^+.

2.2 Photoionization and Recombination of Hydrogen

・electrons produced by photoionization have an initial distribution of energies that depends on $J_\nu a_\nu / h\nu$

↔cross section for elastic scattering collisions between electrons is quite large ($4\pi(e^2/mu^2)^2 \approx 10^{-13}\ \mathrm{cm}^2$)

→collisions tend to set up a Maxwell-Boltzmann energy distribution

・other cross sections involved in the nebulae (including recombination cross section) are much smaller

⇨to a good approximation, the electron-distribution function is Maxwellian

⇨all collisional process occur at rates fixed by local temperature defined by this Maxwellian

⇨recombination coefficient to a specified level n^2L: $\alpha_{n^2L}(\mathrm{H}^0, T) = \int_0^\infty u\sigma_{n^2L}(\mathrm{H}^0, u) f(u)\, du\ [\mathrm{cm}^3\ \mathrm{s}^{-1}]$

・$f(u) = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2} u^2 \exp(-mu^2/kT)$: Maxwell-Boltzmann distribution function for electrons

・σ_{n^2L}: recombination cross section to n^2L in H^0 for electrons with velocity u

・cross sections vary approximately as u^{-2}

⇨recombination coefficient vary approximately as $T^{-1/2}$

2.2 Photoionization and Recombination of Hydrogen

・numerical values of α_{n^2L}: Table 2.1

・mean electron velocities are of order $5 \times 10^7 \mathrm{cm\ s^{-1}}$

⇨recombination cross section are of order 10^{-20} or $10^{-21} \mathrm{cm}^2$

・much smaller than geometrical cross section of H atom

・in nebular approximation discussed previously, recombination quickly leads through downward radiative transitions to 1^2S

→total recombination coefficient is the sum all levels

$$\rightarrow \alpha_A = \sum_{n,L} \alpha_{n^2L}(\mathrm{H}^0, T) \ [\mathrm{cm^3\ s^{-1}}]$$

$$= \sum_n \sum_{L=0}^{n-1} \alpha_{nL}(\mathrm{H}^0, T) = \sum_n \alpha_n(\mathrm{H}^0, T)$$

・α_n: recombination coefficient to all the levels with principal quantum number n

・typical recombination time:

・$\tau_r = 1/n_e\alpha_A \approx 3 \times 10^{12}/n_e \mathrm{\ s} \approx 10^5/n_e \mathrm{\ yr}$

・deviations from ionization equilibrium are decrease in time of this order

Table 2.1

Recombination coefficients (in $\mathrm{cm^3\ s^{-1}}$) $\alpha_n\ {}^2L$ for H

	5,000 K	10,000 K	20,000 K
$\alpha_{1\ ^2S}$	2.28×10^{-13}	1.58×10^{-13}	1.08×10^{-13}
$\alpha_{2\ ^2S}$	3.37×10^{-14}	2.34×14^{-14}	1.60×10^{-14}
$\alpha_{2\ ^2P^o}$	8.33×10^{-14}	5.35×10^{-14}	3.24×10^{-14}
$\alpha_{3\ ^2S}$	1.13×10^{-14}	7.81×10^{-15}	5.29×10^{-15}
$\alpha_{3\ ^2P^o}$	3.17×10^{-14}	2.04×10^{-14}	1.23×10^{-14}
$\alpha_{3\ ^2D}$	3.43×10^{-14}	1.73×10^{-14}	9.49×10^{-15}
$\alpha_{4\ ^2S}$	5.23×10^{-15}	3.59×10^{-15}	2.40×10^{-15}
$\alpha_{4\ ^2P^o}$	1.51×10^{-14}	9.66×10^{-15}	5.81×14^{-15}
$\alpha_{4\ ^2D}$	1.90×10^{-14}	1.08×10^{-14}	5.68×10^{-15}
$\alpha_{4\ ^2F^o}$	1.09×10^{-14}	5.54×10^{-15}	2.56×10^{-15}
$\alpha_{10\ ^2S}$	4.33×10^{-16}	2.84×10^{-16}	1.80×10^{-16}
$\alpha_{10\ ^2G}$	2.02×10^{-15}	9.28×10^{-16}	3.91×10^{-16}
$\alpha_{10\ ^2M}$	2.7×10^{-17}	1.0×10^{-17}	4.0×10^{-18}
α_A	6.82×10^{-13}	4.18×10^{-13}	2.51×10^{-13}
α_B	4.54×10^{-13}	2.59×10^{-13}	1.43×10^{-13}

2.3 Photoionization of a Pure Hydrogen Nebula

・consider simple problem of a single star that is a source of ionizing photons in a homogeneous static cloud of H

- only radiation $(\nu \geq \nu_0)$ is effective in the photoionization of H from the ground level
- ionization equilibrium: $n(\mathrm{H}^0) \int_{\nu_0}^{\infty} \frac{4\pi J_\nu}{h\nu} a_\nu \, d\nu = n_p n_e \alpha_A(\mathrm{H}^0, T)$ $[\mathrm{cm}^{-3}\ \mathrm{s}^{-1}]$ (2.8)

・equation of transfer for radiation $(\nu \geq \nu_0)$: $\frac{dI_\nu}{ds} = -n(\mathrm{H}^0) a_\nu I_\nu + j_\nu$

- I_ν: specific intensity of radiation
- j_ν: local emission coefficient

2.3 Photoionization of a Pure Hydrogen Nebula

- divide the radiation field into two parts
  - ”stellar” part: result directly from the input radiation from the star
  - ”diffuse” part: result from the emission of the ionized gas

  →$I_\nu = I_{\nu s} + I_{\nu d}$
- stellar radiation decreases outward because of geometrical dilution and absorption
  - only source is the star

  →stellar radiation: $4\pi J_{\nu s} = \pi F_{\nu s}(r) = \pi F_{\nu s}(R)\frac{R^2 \exp(-\tau_\nu)}{r^2}$ $[\mathrm{erg\,cm^{-2}\,s^{-1}\,Hz^{-1}}]$ (2.11)
    - $\pi F_{\nu s}(r)$: flux of stellar radiation at r
    - $\pi F_{\nu s}(R)$: flux at R (radius of the star)
    - $\tau_\nu(r) = \int_0^r n(\mathrm{H}^0, r')a_\nu\,dr'$: radial optical depth at r
      - $\tau_\nu(r) = \frac{a_\nu}{a_{\nu_0}}\tau_0(r)$ (2.12) (τ_0: optical depth at the threshold)

2.3 Photoionization of a Pure Hydrogen Nebula

・equation of transfer for the diffuse radiation $I_{\nu d}$: $\frac{dI_{\nu d}}{ds} = -n(\mathrm{H}^0)a_\nu I_{\nu d} + j_\nu$

・for $kT \ll h\nu_0$, the only source of ionizing radiation is recaptures of electrons from the continuum to the ground level

→emission coefficient:

$$j_\nu(T) = \frac{2h\nu^3}{c^2}\left(\frac{h^2}{2\pi mkT}\right)^{3/2} a_\nu \exp[-h(\nu-\nu_0)kT]n_p n_e \ [\mathrm{erg\ cm^{-2}\ s^{-1}\ Hz^{-1}\ sr^{-1}}]\ (\nu > \nu_0)$$

・strongly peaked to $\nu = \nu_0$ (threshold)

・total number of photons generated by recombinations to the ground level:

$$4\pi\int_{\nu_0}^{\infty}\frac{j_\nu}{h\nu}d\nu = n_p n_e \alpha_1(\mathrm{H}^0, T)\ [\mathrm{cm^{-3}\ s^{-1}}]\ (2.15)$$

・$\alpha_1 = \alpha_{1s} < \alpha_A$

→diffuse field $j_{\nu d}$ is smaller than $j_{\nu s}$ on the average

・for optically thin nebula, $j_{\nu d} \approx 0$ as a good approximation

2.3 Photoionization of a Pure Hydrogen Nebula

・for optically thick nebula, good first approximation is that no ionizing photons can escape
⇨diffuse radiation field photon generated in nebula is absorbed elsewhere in the nebula
→ $4\pi \int \frac{j_\nu}{h\nu} dV = 4\pi \int n(\mathrm{H}^0) \frac{a_\nu J_{\nu d}}{h\nu} dV$ (integration is over the entire volume of the nebula)
・"on the spot" approximation amounts to assuming that a similar relation holds locally
→ $J_{\nu d} = \frac{j_\nu}{n(\mathrm{H}^0) a_\nu}$
・exact if all photons were absorbed very close to the point where they are generated ("on the spot")
・diffuse radiation field photons have $\nu \approx \nu_0$
⇨they have large α_ν and correspondingly small mean free paths before absorption
⇨not a bad approximation

2.3 Photoionization of a Pure Hydrogen Nebula

・on-the-spot approximation, (2.11), (2.15), (2.8)

→ionization Equation: $\frac{n(\mathrm{H}^0)R^2}{r^2}\int_{\nu_0}^{\infty}\frac{\pi F_\nu(R)}{h\nu}a_\nu \exp(-\tau_\nu)\,d\nu = n_p n_e \alpha_B(\mathrm{H}^0, T)$ (2.18)

・$\alpha_B(\mathrm{H}^0, T) = \alpha_A(\mathrm{H}^0, T) - \alpha_1(\mathrm{H}^0, T) = \sum_2^{\infty}\alpha_n(\mathrm{H}^0, T)$

⇨in optically thick nebulae, the ionizations caused by stellar radiation-field photons are balanced by recombinations to excited levels of H

・recombinations to the ground level generate ionizing photons that are absorbed elsewhere in the nebula, but have no effect on the overall ionization balance

2.3 Photoionization of a Pure Hydrogen Nebula

- for stellar input spectrum $\pi F_\nu(R)$, left-hand side of (2.18) can be tabulated as a function of τ_0
  - a_ν, τ_ν are known function of ν

→for assumed density distribution, $n_{\mathrm{H}}(r) = n(\mathrm{H}^0, r) + n_p(r)$, temperature distribution $T(r)$, (2.18), (2.12) can be integrated outward

→find $n(\mathrm{H}^0, r)$, $n_p(r) = n_e(r)$

- two models for homogeneous nebulae with $n(\mathrm{H}) = 10\mathrm{H\ cm}^{-3}$ (atom+ion) & $T = 7{,}500K$
  - $\pi F_\nu(R)$ is a blackbody spectrum
    - represent approximately O7.5 main-sequence star
  - $\pi F_\nu(R)$ is a computed model stellar atmosphere

→expected nearly complete ionization out to critical radius r_1

- at r_1, the ionization drops off abruptly to nearly zero

⇨central ionized zone: "HII region" ("H^+ region")

- surrounded by outer neutral H^0 region ("HI region")

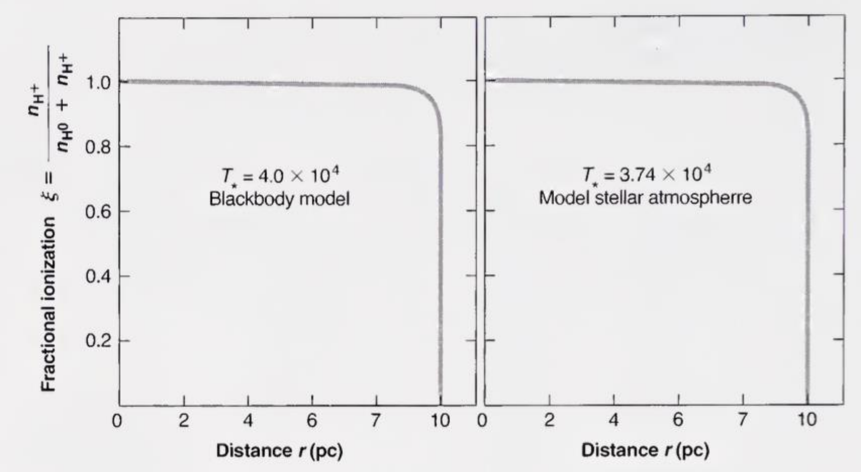

Figure 2.3
Ionization structure of two homogeneous pure-H model H II regions.

2.3 Photoionization of a Pure Hydrogen Nebula

・r_1 can be found from (2.18), (2.12)($\frac{d\tau_\nu}{dr} = n(\mathrm{H}^0)a_\nu$)

$\rightarrow R^2 \int_{\nu_0}^{\infty} \frac{\pi F_\nu(R)}{h\nu} d\nu \int_0^{\infty} d[-\exp(-\tau_\nu)] = \int_0^{\infty} n_p n_e \alpha_B r^2\, dr =$
$R^2 \int_{\nu_0}^{\infty} \frac{\pi F_\nu(R)}{h\nu} d\nu$

・within r_1, ionization is nearly complete ($n_p = n_e \approx n(\mathrm{H})$)

・outside r_1, ionization is nearly zero ($n_p = n_e \approx 0$)

$\rightarrow 4\pi R^2 \int_{\nu_0}^{\infty} \frac{\pi F_\nu}{h\nu} d\nu = \int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu} d\nu = \mathrm{Q}(\mathrm{H}^0) = \frac{4\pi}{3} r_1^3 n_{\mathrm{H}}^2 \alpha_B$

・$L_\nu = 4\pi R^2 \pi F_\nu$: luminosity of the star at frequency ν (per time, unit frequency interval)

⇨total number of ionizing photons emitted by the star balances the total number of recombinations to excited levels within the ionized volume $4\pi r_1^3$ (Strömgren sphere)

・numerical values of radii (Table2.3)→Chapter5

Table 2.3

Calculated Strömgren radii as function of spectral types spheres

Spectral type	T_* (K)	M_V	log $Q(\mathrm{H}^0)$ (photons/s)	log $n_e n_p r_1^3$ n in cm^{-3}; r_1 in pc	log $n_e n_p r_1^3$ n in cm^{-3}; r_1 in pc	r_1 (pc) $n_e = n_p = 1$ cm^{-3}
O3 V	51,200	−5.78	49.87	49.18	6.26	122
O4 V	48,700	−5.55	49.70	48.99	6.09	107
O4.5 V	47,400	−5.44	49.61	48.90	6.00	100
O5 V	46,100	−5.33	49.53	48.81	5.92	94
O5.5 V	44,800	−5.22	49.43	48.72	5.82	87
O6 V	43,600	−5.11	49.34	48.61	5.73	81
O6.5 V	42,300	−4.99	49.23	48.49	5.62	75
O7 V	41,000	−4.88	49.12	48.34	5.51	69
O7.5 V	39,700	−4.77	49.00	48.16	5.39	63
O8 V	38,400	−4.66	48.87	47.92	5.26	57
O8.5 V	37,200	−4.55	48.72	47.63	5.11	51
O9 V	35,900	−4.43	48.56	47.25	4.95	45
O9.5 V	34,600	−4.32	48.38	46.77	4.77	39
B0 V	33,300	−4.21	48.16	46.23	4.55	33
B0.5 V	32,000	−4.10	47.90	45.69	4.29	27
O3 III	50,960	−6.09	49.99	49.30	6.38	134
B0.5 III	30,200	−5.31	48.27	45.86	4.66	36
O3 Ia	50,700	−6.4	50.11	49.41	6.50	147
O9.5 Ia	31,200	−6.5	49.17	47.17	5.56	71

Note: $T = 7{,}500$ K assumed for calculating α_B.