AGNAGN Seminar Sec5.7-5.9

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5.7 Electron Temperatures and Densities from Infrared Emission Lines

・far-infrared lines (such as [OIII] $^3P_0 - {}^3P_1\ \lambda 88\mu\mathrm{m}$ and $^3P_1 - {}^3P_2\ \lambda 52\mu\mathrm{m}$)have much smaller excitation potentials than optical lines (such as $^3P_2 - {}^1D_2\ \lambda 5007$)

- ・ratio like $j_{\lambda 5007}/j_{\lambda 88\mu\mathrm{m}}$ depends strongly on temperature (j: emission coefficient)
  - ・since the $^3P_2$ level has a much lower critical electron density than $^1D_2$, the ratio depends on density
- ・ $j_{\lambda 52\mu\mathrm{m}}/j_{\lambda 88\mu\mathrm{m}}$ hardly depends on temperature at all
  - ・both excitation potentials are so low compared to typical nebular temperatures
  - ・but depend strongly on density

⇨ **by measuring two [O III] ratios, we can determine the average values of T and n_e**

- ・agree reasonably well with values determined from optical lines alone (Figure5.12)

・Including the infrared lines makes determinations of T and n_e possible for many more ions

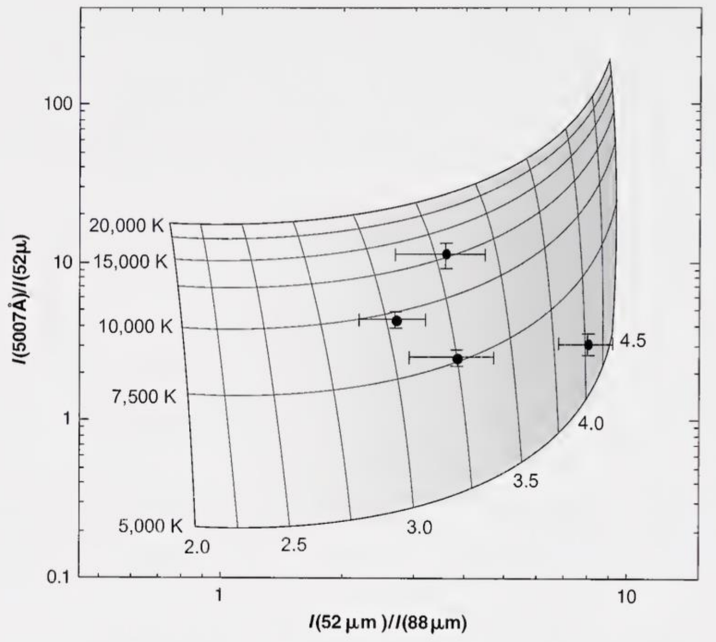

Figure 5.12

Calculated variation of [O III] forbidden-line relative intensity ratios as functions of T (5,000 K to 20,000 K) and the log of the electron density n_e. Observed planetary-nebula ratios plotted with an indication of probable errors.

5.8. Electron Temperatures and Densities from Radio Recombination Lines

- information on T and n_e in gaseous nebulae can be obtained from measurements of the radio recombination lines
  - populations of the high levels of H depend on T and n_e
  - the strengths of the lines emitted by these levels depend on n_e, T, and the optical depth

⇨**comparison of measured and calculated relative strengths can be used to calculate mean values of n_e, T, and E**

- to calculate the expected strengths, solve the equation of radiative transfer
  - since the maser effect is often important
  - continuum radiation isn't weak compared to the line radiation and must be included in the equation of transfer
- observations are reported in terms of brightness temperature
  - T_C: the measured temperature in the continuum near the line
  - T_L: the excess brightness temperature due to the line
  - $T_L + T_C$: the measured brightness temperature at the peak of the line

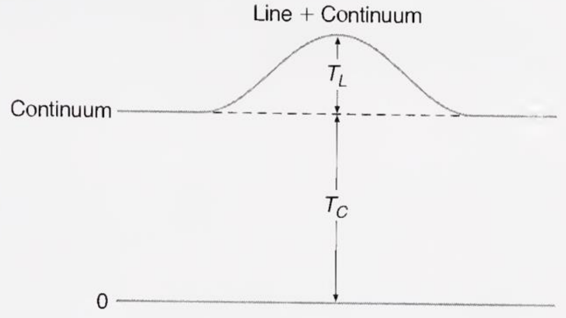

Figure 5.13

A radio-frequency line superimposed on the radio-frequency continuum, showing the brightness temperatures at the center of the line and in the nearby continuum; T_L and T_C, respectively.

Optical depth from radio recombination lines

- consider an idealized homogeneous isothermal nebula
  - optical depth in the center of the line: $\tau_{cL} = \tau_L + \tau_C$
    - τ_C: optical depth in the continuum
    - τ_L: optical depth contributed from the line alone $d\tau_L = \kappa_L ds$ $\kappa_L = n_n k_{0L}.$
      - the central line-absorption cross section

$$\kappa_{0L} = \frac{\omega_m}{\omega_n} \frac{\lambda^2}{8\pi^{3/2}\Delta\nu_D} A_{m,n} \left[1 - \frac{b_m}{b_n} \exp(-h\nu/kT)\right]$$
$$= \frac{\omega_m}{\omega_n} \frac{\lambda^2 (\ln 2)^{1/2}}{4\pi^{3/2}\Delta\nu_L} A_{m,n} \left[1 - \frac{b_m}{b_n} \exp(-h\nu/kT)\right]$$

- $\Delta\nu_D$: the half width at e^{-1} of maximum intensity
- $\Delta\nu_L$: the full-width at half-maximum intensity
- $A_{m,n}$: mean transition probability

correction for stimulated emission

- $$n_m = b_n n^2 \left(\frac{h^2}{2\pi mkT}\right)^{3/2} \exp(X_n/kT) n_p n_e \ [\mathrm{cm}^{-3}]$$
- $\exp(X_n/kT) \approx 1$: good approximation for all observed radio-frequency recombination lines

⇨express $A_{m,n}$ in terms of the corresponding f-value ($f_{m,n}$) and expand the stimulated-emission correction for the case of local thermodynamic equilibrium ($b_m = b_n = 1$, denote by *)

→
$$\tau_L^* = 1.53 \times 10^{-9} \frac{n^2 f_{nm} \nu}{\Delta\nu_L T^{2.5}} E_p$$
$$= 1.01 \times 10^7 \frac{\Delta n \ f_{nm}}{n \Delta\nu_L T^{2.5}} E_p.$$

- proton-emission measure (E_p): $E_p = \int n_p n_e \, ds$
- $\nu = \frac{\nu_0}{n^2} - \frac{\nu_0}{m^2} \approx \frac{2\nu_0 \Delta n}{n^3}$

Temperature from Radio Recombination Lines

・In the true nebular case

・the line optical depth:

$$\tau_L = \tau_L^* b_n \frac{\left[1 - \frac{b_m}{b_n} \exp(-h\nu/kT)\right]}{\left[1 - \exp(-h\nu/kT)\right]}$$
$$= \tau_L^* b_m \left(1 - \frac{kT}{h\nu} \frac{d \ln b_n}{dn} \Delta n\right)$$

・the continuum optical depth is the same as in thermodynamic equilibrium

・the free electrons have a Maxwellian distribution

⇨calculate the ratio of brightness temperatures ($r = T_L/T_C$) in case of thermodynamic equilibrium

→
$$r^* = \frac{T_L + T_C}{T_C} - 1 = \frac{T[1 - \exp(-\tau_{CL})]}{T[1 - \exp(-\tau_C)]} - 1$$
$$= \frac{1 - \exp[-(\tau_L^* + \tau_C)]}{1 - \exp(-\tau_C)} - 1.$$

⇨
$$r^* = \frac{\tau_L^*}{\tau_C}.$$

・$\tau_L^* \ll 1$: good approximation in all lines observed to date

・$\tau_C \ll 1$: generally but not always a good approximation

⇨ under the assumption of local thermodynamic equilibrium, the observed ratio of brightness temperatures (r) gives the ratio of optical depths

→measure T

Temperature from Radio Recombination Lines

- calculate the brightness-temperature ratio $r = T_L/T_C$ in the true nebular in the case of non thermodynamic equilibrium
  - brightness temperature in the continuum: $T_C = T[1 - \exp(-\tau_C)]$.
  - both the line-emission and line-absorption coefficients differ from their thermodynamic equilibrium values
    - line-emission coefficient: $j_L = j_L^* b_m$,
      - depends on the population in the upper level
    - line-absorption coefficient

$$\kappa_L = \kappa_L^* b_m \beta, \qquad \beta = 1 - \frac{kT}{h\nu}\frac{d \ln bn}{dn}\Delta n.$$

  - equation transfer in intensity units:

$$\frac{dI_\nu}{d\tau_{CL}} = -I_\nu + \frac{j_L + j_C}{\kappa_L + \kappa_C} = -I_\nu + S_\nu,$$

$$S_\nu = \frac{j_L^* b_m + j_C}{\kappa_L^* b_m \beta + \kappa_C} = \frac{\kappa_L^* b_m + \kappa_C}{\kappa_L^* b_m \beta + \kappa_C} B_\nu(T)$$

⇨ brightness temperature at the center of the line:

$$T_L + T_C = \left[\frac{\kappa_L^* b_m + \kappa_C}{\kappa_L^* b_m \beta + \kappa_C}\right] T \left\{1 - \exp\left[-\left(b_m \beta \tau_L^* + \tau_C\right)\right]\right\}$$

⇨

$$r = \frac{T_L}{T_C} = \left[\frac{\kappa_L^* b_m + \kappa_C}{\kappa_L^* b_m \beta + \kappa_C}\right]\left[\frac{1 - \exp\left(-b_m \beta \tau_L^* + \tau_C\right)}{1 - \exp\left(-\tau_C\right)}\right] - 1,$$

- depends only on one optical depth (τ_C), the ratio of optical depths, ($\tau_L^* \tau_C = \kappa_L^* \kappa_C$), and the b_n factors

⇨**r depends on n_e and T**

⇨**when the deviations from thermodynamic equilibrium are taken into account, r depends on T, n_e, τ_C, and E**

determination T, n_e, and E

⇨**observations of several different lines in the same nebula are necessary to determine T, n_e, and E**

・**make the best possible match between all measured lines in a given nebula, and the theoretical calculations for a given T, n_e,and E**

↮observational problems

・the radio recombination lines suffer significant impact broadening at the low densities of nebulae

⇨wings of the line difficult to define observationally

・measurements are made at different frequencies and with different radio telescopes

⇨the antenna beam patterns are not identical for all lines

・from model calculation

・measurements of lines with $\Delta n = 1$ at frequencies near 10 GHz are only slightly affected by maser effects and by deviations from thermodynamic equilibrium

⇨suitable for determining nebular temperatures

・Figure5.14: compare the temperature from recombination lines and from collisionally excited lines

・temperatures determined from the recombination lines tend to be lower

・ also found with the Balmer jump and radio continuum

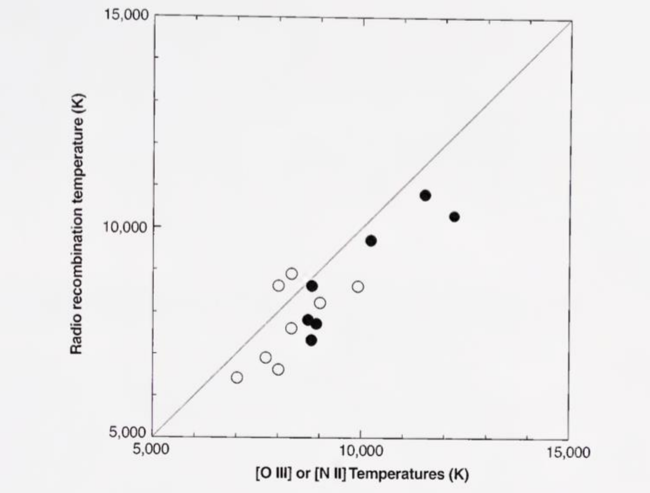

Figure 5.14
A comparison between temperatures determined from optical forbidden lines ([O III] filled circles; [N II] open circles) and radio recombination lines.

determination T, n_e, and E

- range in temperatures is largely due to physical differences among the nebulae
  - the measured temperatures shows increase with increasing distance from the center
    - consistent with the decrease in heavy-element abundance outward from the center (more detail in Capter10)

- average electron densities from the radio recombination line measurements
  - **best way is to compare lines of two different frequencies**
    - important to match the antenna beam widths
    - very high n lines cannot be used
      - because impact broadening becomes important, making the wings difficult to define

  →mean electron density derived in this way for the Orion Nebula: $n_e = 2.4 \times 10^3\ \mathrm{cm}^{-3}$
      - comparable with an emission-weighted average of the [O II] determinations

5.9 Filling and Covering Factors

・chaotic structure in planetaries and H II regions

・density condensations, low-density hollows, etc.

→ important feature of the structure of gaseous nebulae

・these structures can be detected quantitatively if the densities derived from [O II] line ratios are used to predict the expected high-frequency radio continuum brightness temperature

・ in this case

→brightness temperature: $T_{b\nu} = 8.24 \times 10^{-2} T^{-0.35} \nu^{-2.1} E_c$

・in the limit of small optical depth (a good approximation for high-frequency observations)

・E_C: continuum emission measure ($E_C = \int n_+ n_e \, ds$)

・predicted brightness temperature depends only very weakly on the nebular temperature

Filling and Covering Factors

→**measured values of $T_{b\nu}$ are smaller than predicted value** in this way, typically ×10

・can be understood if the nebula is thinner along the line of sight than perpendicular to it, or density fluctuations

・line-ratio density measurements are heavily weighted toward the regions of strongest emission

⇨measured densities deviate from the average density along a typical path or ray through the nebula

⇨**density fluctuation must be taken into account in describing the structure of the nebula**

・simplest way: idealize the nebula as containing gas, in small clumps or condensations with n_e or between them with zero electron density

・filling factor (ε): the fraction of the total volume occupied by the condensations

・space between the condensations is a vacuum with no contribution to the emission, mass, opacity, etc.

・can be assumed to be constant throughout a nebula or can be allowed to vary with position

・covering factor ($\Omega/4\pi$): fraction of 4π sr that is covered by gas, as viewed from the central star

・takes into account regions where the gas is nonexistent or has insufficient column density to fully absorb the ionizing continuum (appear fainter)

under filling and covering-factor description of nebulae

→intensity of an emission: $I_l = \int j_l \, ds = \int \varepsilon \, n_i n_e \varepsilon_l(T) \, ds$:

→luminosity in the same line, integrated over the volume of the nebula: $L_l = \frac{\Omega}{4\pi} \int \varepsilon \, n_i n_e \varepsilon_l(T) \, dT$

→other properties can be determined

- the number of recombinations, the total mass of H in the nebula, the radial optical depth

- Similar generalizations can be made in other equations
  - on the basis that n describes the density in the condensations with vacuum between them