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Section 2.4 – End of Section 2

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2.4 Photoionization of a Nebula Containing Hydrogen and Helium

Taking both H and He elements: better ionization structure of an actual nebula

- ionization potential of He = 24.6 eV (higher than H)
- ionization potential of He+ = 54.4 eV
    - hottest O stars emit practically no photons with $h\nu > 54.4$ eV -> second ionization of He does not occur in ordinary H II regions

photons with energy $13.6 \text{ eV} < h\nu < 24.6$ eV: ionize H only
photons with energy $h\nu > 24.6$ eV: ionize both H and He

➔ **Two different types of ionization structure are possible depending on**
- **spectrum energy of ionizing radiation**
- **abundance of He**

At one extreme example)
spectrum is concentrated to frequencies just above 13.6 eV + only a few photons with $h\nu > 24.6$ eV
➔ the photons with energy $13.6 \text{ eV} < h\nu < 24.6$ eV keep the H ionized, and the photons with $h\nu > 24.6$ eV are all absorbed by He.
➔ ionization structure:
- small central H^+, He^+ zone
- surrounded by a larger H^+, He^0(=He) region.

$a_\nu(He^0)$: He^0 photoionization cross section

The photoionization cross section for the $1\,^2S$ level of H^0, or, in general, of a hydrogenic ion with nuclear charge Z, may be written in the form

$$a_\nu(Z) = \frac{A_0}{Z^2}\left(\frac{\nu_1}{\nu}\right)^4 \frac{\exp\{4-[(4\tan^{-1}\varepsilon)/\varepsilon]\}}{1-\exp(-2\pi/\varepsilon)}\ [\mathrm{cm}^2]\ \text{for } \nu \geq \nu_1 \qquad (2.4)$$

where

$$A_0 = \frac{2^9\pi}{3e^4}\left(\frac{1}{137.0}\right)\pi a_0^2 = 6.30\times10^{-18}\ [\mathrm{cm}^2],$$

$$\varepsilon = \sqrt{\frac{\nu}{\nu_1}-1},$$

and

$$h\nu_1 = Z^2 h\nu_0 = 13.6Z^2\ \mathrm{eV}$$

Photoionization cross section (σ): probability that a photon with a specific energy will ionize an atom or ion, expressed per unit area.

The larger this cross-sectional area results in a greater likelihood of photoionization.

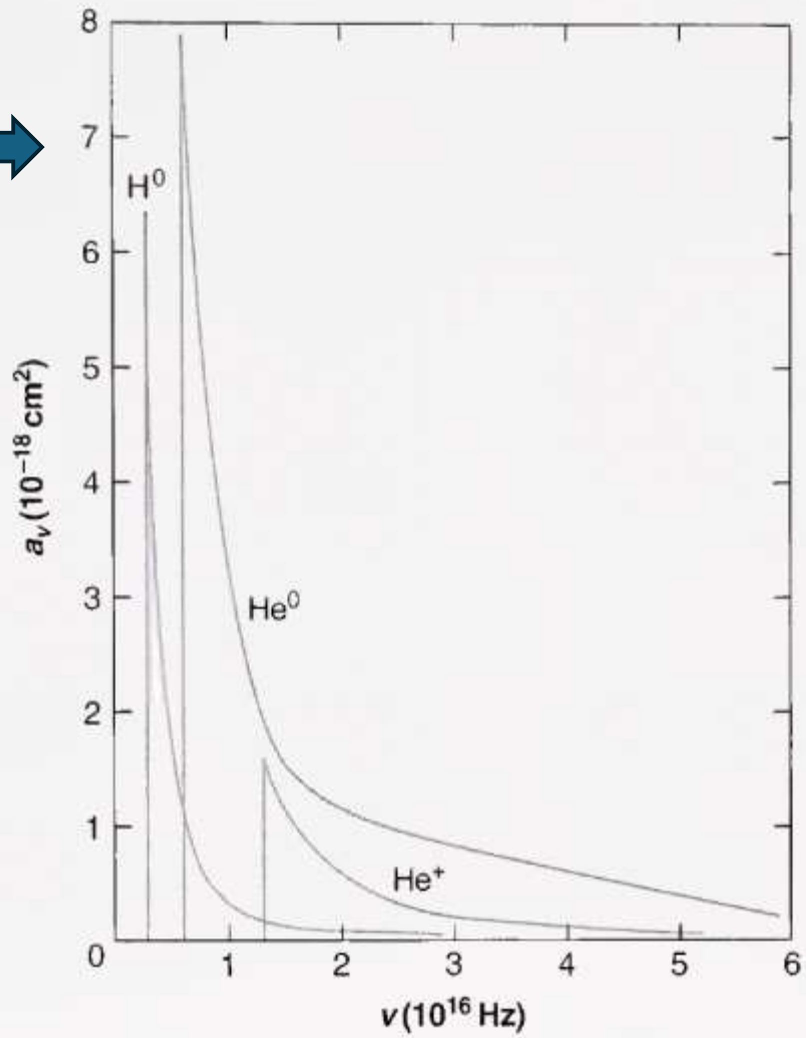

Figure 2.2
Photoionization absorption cross sections of H^0, He^0, and He^+.

2.4 Photoionization of a Nebula Containing Hydrogen and Helium

Helium: two-electron system -> singlet/triplet splitting, metastable states, and collision-induced transitions -> exhibits more complex ionization and recombination behaviors

1. Photoionization Cross Section of He^0 Compared
- Photons with sufficient energy (especially $h\nu > 24.6$ eV): ionize both He^0 and H^0
- How much these photons are absorbed by H vs He:
  - depends on their **relative photoionization cross sections** and **densities**

2. He Becomes Hydrogen-like at Higher Levels (L > 2)
- Electrons in high-energy levels (L > 2) of He behave similarly to those in H
  - the **recombination coefficient (α)** is nearly the same in this regime.
- **At lower levels (especially S and P terms)**, significant differences arise.

3. Reason: Singlet & Triplet States in He
- He has two electrons
  - its total spin leads to singlet (S = 0) and triplet (S = 1) states
- This causes the **recombination coefficient α** for He to differ from that of H, particularly for:
  - **S term (L = 0)**
  - **P term (L = 1)**

4. Absorption of High-Energy Photons ($h\nu > 24.6$ eV)
- High-energy photons emitted during recombination to He^+ can reionize H^0 or He^0.
- The absorption ratio **y** is defined as:
  - **y**: fraction absorbed by H
  - **1 − y**: fraction absorbed by He

$$y = \frac{n(H^0)\, a_{\nu_2}(H^0)}{n(H^0)\, a_{\nu_2}(H^0) + n(He^0)\, a_{\nu_2}(He^0)},$$

5. Triplet States Dominate in Recombination to He

During recombination from He^+ to He^0:

- About **75%** of recombinations: go to **triplet states** (e.g., 2^3S)
- About **25%** of recombinations: go to **singlet states** (e.g., 2^1S, 2^1P)

6. The 2^3S State is Metastable

This state is long-lived and does not decay easily.

To decay, it must undergo either:

- A **forbidden transition** via emission of a 19.8 eV photon → very slow
- more commonly, **collisional excitation** by electrons to a singlet state

1. Collisional Transitions to Singlet States

The transitions from **$2^3S \rightarrow 2^1S$** or **2^1P** require a spin flip

The probability for **collisional transitions** depends on:

- **σ**: Electron collision cross section
- **f(u)**: Electron energy distribution function

$$n_e\, q(2\,^3S, 2\,^1L) = n_e \int_{\frac{1}{2}mu^2=\chi}^{\infty} u\sigma(2\,^3S, 2\,^1L, u)\, f(u)\,du$$

2. Key Concept: Critical Electron Density (n_c)

There are two possible pathways for transitions from the 2^3S state:

- **Radiative transition** (via forbidden lines)
- **Collisional transition**

dominates depends on the **electron density**.

define the **critical electron density (n_c)** as the density where both processes occur at comparable rates:

$$n_c(2\,^3S) = \frac{A(2\,^3S, 1\,^1S)}{q(2\,^3S, 2\,^1S) + q(2\,^3S, 2\,^1P^o)}$$

3. Electron Density vs. Transition Mechanism (Collisional vs. Radiative)

- Atomic state transitions can occur via two mechanisms:
  - **Radiative transitions**: State changes by emitting a photon
  - **Collisional transitions**: State changes via collisions with other particles
- electron density **(n_e)** < the critical density **(n_c)** → radiative transitions dominate
- $n_e > n_c$ → collisional transitions become significant

4. Examples in Different Astrophysical Environments

- **H II Regions** (typical ionized hydrogen regions):
  - Electron density $n_e < 10^2\ cm^{-3} \rightarrow n_e \ll n_c$ → **Radiative transitions dominate**.
  - The 2^3S state of He atoms typically decays by emitting a 19.8 eV photon.
- **Bright Planetary Nebulae**:
  - Electron density $n_e \sim 10^4\ cm^{-3} \rightarrow n_e > n_c$ → **Collisional transitions become significant**.
  - Before undergoing radiative decay, He atoms are more likely to transition to other states (e.g., 2^3P or 2^1P°)

5. He Level Transitions and Their Consequences

helium atoms cascade down from higher energy levels -> emit photons through various transitions.
Some of these photons can ionize hydrogen atoms.

- For example:
  - $2^1P^\circ \rightarrow 1^1S$: Emits a 21.2 eV photon (a strong resonance line).

    This photon may undergo multiple scatterings by He^0 and eventually either
    - convert into a 2.06 μm line, or
    - ionize a hydrogen atom.

Electron density conditions	p value (H ionization contribution)
low density ($n_e \ll n_c$)	~ 0.96
high density ($n_e \gg n_c$)	~ 0.66

6. Incorporating He Transitions into the Hydrogen Ionization Balance

- On-the-spot approximation: Assumes photons are immediately reabsorbed nearby.
- Under this approximation, the contribution of ionizing photons from He transitions is included via a factor **p** in the hydrogen ionization equation.

7. Variation of p with Electron Density

p: Fraction of photons emitted during He recombination that go on to ionize hydrogen.
As electron density decreases,
→ p increases
→ He-generated photons more effectively ionize H in low-density environments.

2.4 Photoionization of a Nebula Containing Hydrogen and Helium

1. Calculation of the size of the He^+ zone (helium ionization zone)

- exact radius of the He^+ zone: solving complex equations numerically.
- ignore absorption by hydrogen -> it can be expressed by a simple formula:

$$\int_{\nu_2}^{\infty} \frac{L_\nu}{h\nu} d\nu = Q(\mathrm{He}^0) = \frac{4\pi}{3} r_2^3\, n(\mathrm{He}^+)\, n_e \alpha_B(\mathrm{He}^0), \qquad (2.27)$$

2. Calculation of the size of the H^+ zone (hydrogen ionization zone)

- Similarly to the He^+ zone, the calculation can be applied analogously:

$$\int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu} d\nu = Q(\mathrm{H}^0) = \frac{4\pi}{3} r_1^3 n(\mathrm{H}^+) n_e \alpha_B(\mathrm{H}^0). \qquad (2.19)$$

3. Ionization structure depending on the star's temperature

- At higher the star's temperature -> H^+ and He^+ zones nearly coincide.
- At lower temperatures -> H^+ zone is larger while the He^+ zone is more confined, resulting in a separated structure.

4. Summary

- Helium recombination: significant impact on hydrogen ionization.
- ionization zone structure: varies considerably depending on the density and the star's temperature.
- By numerically calculating these processes, the ionization structure of nebulae can be accurately modeled.

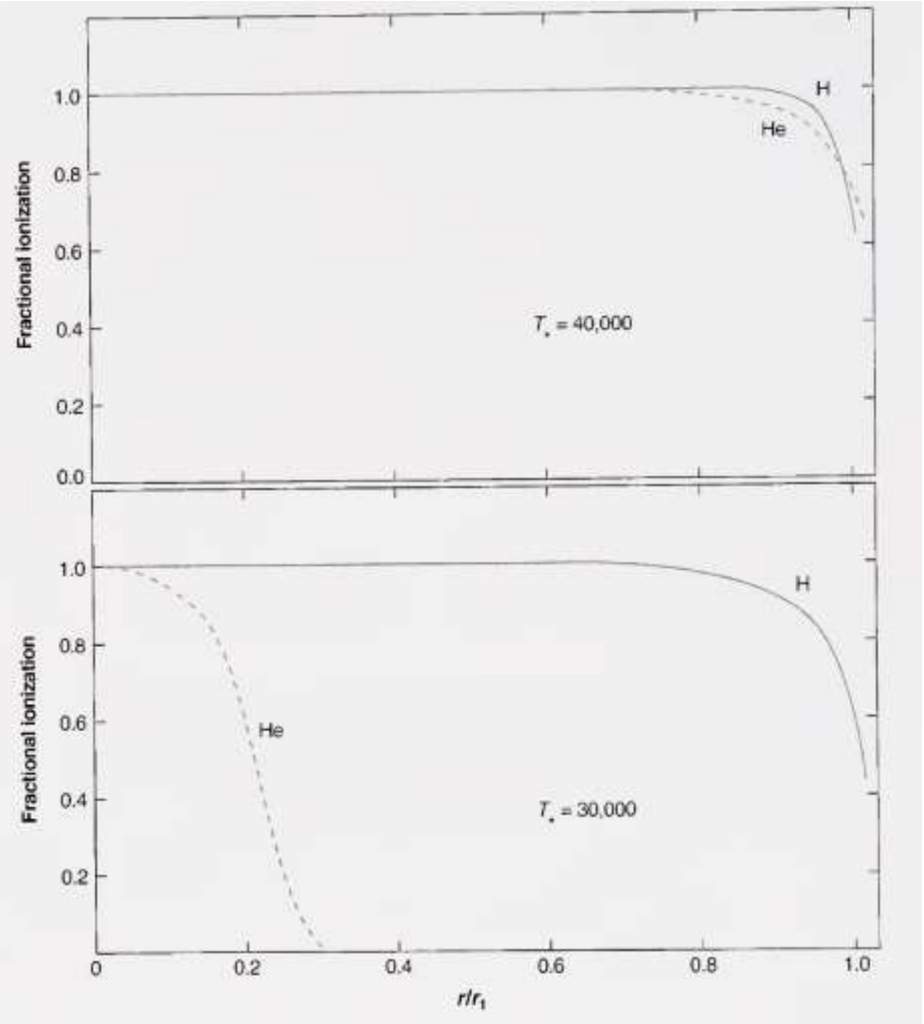

Figure 2.4
Ionization structure of two homogeneous H$^+$ He model H II regions.

2.4 Photoionization of a Nebula Containing Hydrogen and Helium

5. Ionizing photon count ratio formula

$$\left(\frac{r_1}{r_2}\right)^3 = \frac{Q(\mathrm{H}^0)}{Q(\mathrm{He}^0)}\frac{n_{\mathrm{He}}}{n_{\mathrm{H}}}\left(1+\frac{n_{\mathrm{He}}}{n_{\mathrm{H}}}\right)\frac{\alpha_B(\mathrm{He}^0)}{\alpha_B(\mathrm{H}^0)} \quad \text{if } r_2 < r_1. \qquad (2.28)$$

- $Q(He^0)$: The number of high-energy photons that can ionize He^0
- $Q(H^0)$: The number of photons that ionize H^0
- n(He)/n(H): Helium/hydrogen number density ratio
- α_B: recombination coefficient
- r_2: Radius of the He^+ area
- r_1: Radius of the H^+ area

→ equation connects the number of photons, the density ratio of elements, and the volume ratio of the two ionization regions (the cube of the radius).

6. Interpretation of Figure 2.5

how much smaller the He^+ region is compared to the H^+ region, depending on the star's temperature ($T_\star$)

What happens when the temperature increases?

- If T★ is above 40,000 K, the He^+ region and the H^+ region almost coincide → $r2/r1 \approx 1$

This means that if the star is very hot, both helium and hydrogen are ionized at similar locations.

- The lower the T★, the smaller the He^+ region becomes

This means that the cooler the star, the less high-energy photons are needed to ionize helium.

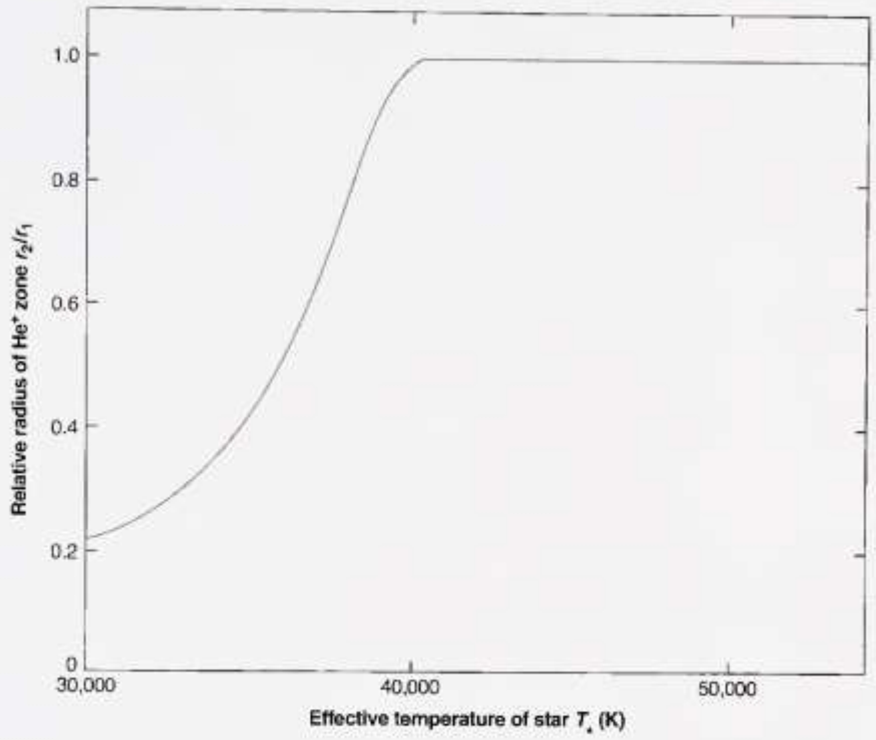

Figure 2.5
Relative radius of He^+ zone as a function of effective temperature of exciting star.

2.5 Photoionization of He^+ to He^{++}

This chapter:

- how helium ions (He^+) are ionized again in a planetary nebula to form He^{2+} (He^{++})
- how the high-energy photons produced thereby contribute to hydrogen ionization (H^+ maintenance)

1. Differences from typical H II regions in galaxies

Typical O-type stars (the central stars of H II regions in galaxies):

- emit photons with energy $h\nu > 54.4$ eV.
- no He^{2+} (He^{++}) zone.

Central stars of planetary nebulae:

- much hotter and emit enough high-energy photons (>54.4 eV).

→ A He^{++} zone forms at the center

→ observed through the He II recombination lines (He II spectrum).

2. The He^{++} zone is also an H^+ zone

- He^{++} zone is formed by further ionization of He^+
- hydrogen inside this zone remains ionized → this region is also part of the H^+ zone

3. Photons emitted during He^{++} → He^+ recombination significantly reionize hydrogen

High-energy photons released during recombination strongly contribute to hydrogen reionization

[Three main processes]

1) He II Lyman-alpha (La):

- 2P → 1S transition (energy: 40.8 eV)
- High enough energy to ionize hydrogen

2) Two-photon continuous emission:

- 2S → 1S transition
- Emits two photons simultaneously (total energy 40.8 eV)
- about 1.42 photons capable of ionizing hydrogen are emitted

3) Balmer continuum emission:

- recombines into 2S or 2P states,
- emits many photons just above hydrogen's ionization threshold (13.6 eV).

4. Why these photons are important for maintaining hydrogen ionization

- He II Lyman-alpha and Balmer continuum photons from above continue to ionize H^{0} (neutral hydrogen)
- hydrogen ionization (H^{+}): maintained even within the He++ region
- He^{++} -> He^{+} recombination photons are more important than the 13.6–54.4 eV photons directly emitted by the star to ionize H^{0}.

5. Numerical Example – Table 2.6

Total ≈ 1.2–1.3 H ionization photons are produced per He++ recombination

➔ Enough photons are produced to maintain the H+ state

Type of photon	T = 10,000K	T = 20,000K
He II Lyman-alpha photon count	0.64	0.66
He II two-photon emission	0.36	0.42
Balmer continuous line emission	0.20	0.25

6. Calculating the radius r_3 of the He++ region (Strömgren radius analogous formula)

$$Q\left(\mathrm{He}^{+}\right)=\int_{4\nu_0}^{\infty}\frac{L_\nu}{h\nu}d\nu=\frac{4\pi}{3}r_3^3 n\left(\mathrm{He}^{++}\right)n_e\alpha_B\left(\mathrm{He}^{+},T\right) \tag{2.29}$$

- the same structure as the formula for calculating the Strömgren radius of the H^{+} region
- the star temperature $T_{\star}$ must be greater than 10^{5} K

→ The He++ region almost fills the center of the entire H^{+} region.

2.6 Further Iterations of the Ionization Structure

This chapter:
- the methods used to calculate the ionization structure of nebulae
- the iterations that have been made to improve their accuracy.

1. On-the-spot approximation:
- assumes that the ionizable photons emitted when ions recombine are immediately absorbed by the surroundings
- With this simple assumption, the ionization structure and temperature inside the nebula can be calculated

2. More precise calculations through iterations
increase the precision by iterating as follows:
1) First step (on-the-spot):
Approximately calculate the ionization state and temperature
This gives the emission coefficient j_ν (the emission energy per unit volume at a specific frequency)
2) Second step:
Using j_ν, calculate the radiation field in the nebula
3) Third step:
Using this new J_ν value, recalculate the ionization structure and temperature more precisely
4) This process is repeated to get closer and closer to the accurate result
If enough calculation time is given, it converges quickly (i.e., reaches a stable result quickly)

3. Deeper facts revealed through repetition
- iterative calculations → the ionization degree near the center of the nebula is very high.
- ionization photons created by recombination are not absorbed immediately, but spread outward.
  → Ultimately, play a role in helping to ionize the outer part of the nebula.
- the ionization structure obtained by the first approximation is also quite good.

2.7 Photoionization of Heavy Elements

This chapter explains

- how photoionization of heavy elements (oxygen, carbon, nitrogen, etc.) in a nebula occurs.

1. What is photoionization of heavy elements?

- atomic numbers greater than hydrogen (H) or helium (He)
    - Ex) oxygen (O), carbon (C), nitrogen (N), neon (Ne), silicon (Si), and iron (Fe).
- less abundant than hydrogen
- play a significant role in the energy balance and spectrum of a nebula.
- Photoionization is the process in which a high-energy photon strikes an atom and ejects an electron from it.

2. Ionization Equilibrium

- The balance between the *i-th* and *i+1-th* ionization states of an element X can be expressed as:
- **Photoionization rate = Recombination rate**

$$n(X^{+i}) \int_{\nu_i}^{\infty} \frac{4\pi J_\nu}{h\nu} a_\nu(X^{+i}) d\nu = n(X^{+i})\Gamma(X^{+i}) \tag{2.30}$$
$$= n(X^{+i+1}) n_e \alpha_G(X^{+i}, T),$$

- **Left-hand side**: The rate at which the *i-th* ionization state (X^{+i}) is ionized by photons
- **Right-hand side**: The rate at which the *(i+1)-th* state ($X^{+(i+1)}$) recombines with electrons to return to the *i-th* state

Symbol	Meaning
X^{+i}	The ith ionization state of element X (e.g. O^{+}, O^{2+}, etc.)
$n(X^{+i})$	Density of the i-th ionization state of X (cm^{-3})
J_ν	The average intensity of the radiation field at a specific frequency (ν)
$\alpha_\nu(X^{+i})$	Photoionization cross section (the probability that a photon at that energy will remove an electron)
$\alpha_G(X^{+i},T)$	Recombination coefficient (probability of electrons joining), temperature T dependent
n_e	Density of free electrons
$h\nu$	The energy of a photon

3. Conservation of Total Atomic Number

For any element X, the sum of all ionization steps must be equal to the total number of X atoms:

$$n(X^0) + n(X^+) + n(X^{++}) + \ldots = n(X)$$

4. Effect of radiation fields

- amount of heavy elements is small -> do not play a significant role in forming or modifying the radiation field itself.
- sufficient to consider only the radiation field (J_ν) from H and He

5. Contribution to optical thickness

- heavy elements do not contribute significantly to the overall optical thickness
- under certain conditions, they can contribute, especially near the He^0 ionization critical energy.

6. Inner Shell Ionization and X-Rays

- Heavy elements have inner shells in addition to their outer shells
- High-energy photons can also knock away these inner electrons, which:
    - High-energy emission (X-rays)
    - Auger Effect: the process in which one electron is removed and another is ejected.

7. Characteristics of ionization cross section (σ)

- photoionization cross section of heavy elements:
    - large near the critical energy
    - decreases as the energy increases
- unlike hydrogen, strong resonance structures can be formed due to the interaction between electrons

8. Types of recombination coefficients

recombination coefficients of heavy elements are broadly divided into two types:

$\alpha_G = \alpha_R + \alpha_D$

- α_R: Radiative recombination (electrons recombine and emit photons)
- α_D: Quasi-electronic recombination (a complex process involving the interaction of electrons and electrons)

9. Differences with hydrogen-type atoms

- Many recombinations are simple, like hydrogen
- heavier elements with complex electron configurations behave differently than hydrogen
    - For example) Pauli exclusion principle -> recombination to some energy levels is suppressed or complicated

10. Two Important Processes in the Ionization Equilibrium of Heavy Elements

1) Radiative Recombination

- a free electron recombines with an ion and emits a photon.
- typical and most common type of recombination.

2) Dielectronic Recombination

- where a free electron is captured by an ion while simultaneously exciting another bound electron within the ion.
- temporary **doubly-excited** (resonant) state.
- highly efficient under certain physical conditions.

2-1) Detailed Mechanism of Dielectronic Recombination

[Mechanism]

1. A free electron approaches an ion (e.g., X^{++}).
2. Most of the electron's energy is used to excite another bound electron in the ion
3. If the system stabilizes (without autoionization), the electron becomes bound, completing the recombination.
4. The ion then transitions through several radiative steps to return to the ground state.

2-2) Why is Dielectronic Recombination Important?

- very big effect when it is in resonance condition
- becomes more important as the temperature increases (more than several thousand to ten thousand K).

3) Another important process: Charge Exchange

: A two-particle reaction in which two atoms exchange electrons with each other, changing their ionization state.

- Example) $O^0 + H^+ \rightarrow O^+ + H^0$

Neutral oxygen (O^0) meets proton (H^+) and ionizes into O^+

This reaction:

- relatively large cross-section (efficient)
- near resonance condition → small energy difference
- very important in places where neutral hydrogen (H^0) is abundant, such as the outer nebula